E.M. Lifshits, E.L. Andronikashvili

A Supplement To
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TRANSLATED FROM RUSSIAN

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FROM THE PUBLISHER

In 1942, a book, *Helium,* by Professor W.H. Keesom, was published by Elsevier Publishing Company. This book rapidly won recognition as a classic in the field of low temperature physics. It is currently out of print.

In 1949, The Academy of Sciences of the USSR published a Russian translation of Professor Keesom's book. This Russian edition, of which the late Professor Keesom was never aware, contained two supplementary chapters; "Superfluidity (Theory)" by E.M. Lifshits and "Superfluidity (Experimental Data)" by E.L. Andronikashvili.

A Supplement to "Helium" comprises the Consultants Bureau translation of these two supplementary chapters to the Russian edition of *Helium* by Professor W.H. Keesom.

The reader will note references to figures, diagrams and tables from Chapters 1-7 of the original work which he may consult. They are not reproduced here.

We wish to express our appreciation to the copyright owners of Professor Keesom's original work — Elsevier Publishing Company — and to Professor Keesom's son — Professor P.H. Keesom, Purdue University — for their gracious permission to refer to *Helium* in the title and in the announcements of *A Supplement to "Helium"*.

CHAPTER VIII

SUPERFLUIDITY (THEORY)[1]

E. M. Lifshits

Section 1. Quantization of the Motion of a Liquid

HELIUM II − A QUANTUM LIQUID. At temperatures below the
λ-point liquid helium (helium II) has a number of surprising pro-
perties which are due to the quantum nature of this liquid.

As the temperature is reduced the de Broglie wavelength cor-
responding to the thermal motion of the atoms is increased. In
liquid helium the de Broglie wavelength of the helium atoms be-
comes comparable with the interatomic distances at temperatures
of the order of 2 - 3°K; as a result the behavior of liquid helium is
intimately related to quantum effects in this temperature region
and in this sense helium at very low temperatures may be called a
"quantum liquid". In particular, the quantum behavior is manifest
in the fact that helium remains a liquid (at ordinary pressures) at
all temperatures down to absolute zero whereas according to clas-
sical physics any substance must become a solid crystal at absolute
zero.

Liquid helium is the only quantum liquid found in nature; all
other liquids solidify long before quantum effects can be noticed.

The most important property of liquid helium is its superfluidity,
an effect which was discovered by Kapitza. An explanation and

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[1]The text of the present chapter has been published, with the permission
of the publisher, as a survey paper (in somewhat different form) in Usp.
Fiz. Nauk 34, 513 (1948).

quantitative theory for this effect was given by Landau [1-3] who first formulated a consistent theory for this quantum liquid. The present chapter is devoted to a presentation of this theory.

Of other attempts to formulate a theory of superfluidity we may mention those of London [4-6] and Tisza [7-8] (the later work of Tisza [9-10] is discussed below in Section 3). In this work an attempt was made to explain the behavior of helium II by invoking the properties of a degenerate ideal Bose-Enstein gas: it was assumed that the atoms in the ground state (state of zero energy) move through the substance without friction. This notion, however, is not satisfactory. First of all, helium II has nothing in common with an ideal gas, and there is no basis for applying results obtained from analyses of a gas. Indeed, in an ideal gas the atoms in the normal state in no way exhibit "superfluidity;" on the contrary, there is nothing to prevent their colliding with excited atoms, exchanging momentum in the process, i.e., these atoms would encounter friction in their motion in the substance, and there would be no superfluidity. This interpretation of superfluidity is not well founded; indeed, its very essence is found to be in direct contradiction with the original assumptions.

ENERGY SPECTRUM OF A QUANTUM LIQUID. The problem of a quantum liquid is essentially that of finding its energy spectrum, i.e., the set of quantum-mechanical energy levels. We wish to emphasize that what we are discussing here are levels corresponding to states of the liquid taken as a whole, not levels of the individual atoms; in a quantum-mechanical system consisting of particles which interact strongly, such as a quantum liquid, the notion of states of the individual atoms is actually not very meaningful.

In general, a complete calculation of the energy spectrum of a real liquid is not possible since it would involve the particular law which governs the interaction between atoms of the liquid. However, from general theoretical considerations it is possible to predict a number of properties of this spectrum, and these are sufficient for predicting the basic characteristics of a quantum liquid.

The behavior of the liquid at very low temperatures is determined by that part of the energy spectrum which lies slightly above the ground level (at which the liquid is at absolute zero). The fol-

lowing statement may be made with respect to this part of the spectrum. Any weakly excited state of a macroscopic body may, in quantum mechanics, be considered as the net result of a number of individual "elementary excitations." These "elementary excitations" can be thought of as "quasi-particles" which move in the occupied volume and have definite energy and momentum characteristics. Thus, for example, the thermal excitation of a solid body (crystal), in which the atoms execute small vibrations about their equilibrium positions, may be considered, as is well known, in terms of "acoustic quanta" (phonons) which move through the body.

According to Landau there are two basic types of energy spectra for macroscopic quantum systems; these may be called the Fermi type and the Bose type.

A Bose spectrum is characterized by the fact that in excitation of the body the excitation quanta appear one at a time. On the other hand, the angular momentum of any quantum-mechanical system (in the present case − the entire liquid) can change only by whole numbers. Hence the elementary excitations, which appear one at a time, should have integral momentum and thus, obey Bose statistics. Such, for example, is the phonon spectrum for thermal vibrations of atoms in a crystal and the Bloch spin-wave spectrum in a ferromagnet.

On the other hand, the Fermi spectrum is characterized by the fact that the elementary excitations must appear in pairs in the excitation of the body. The constituents of such a pair have certain orooerties which are "supplementary" with respect to each other ("particle" and "hole") and which manifest themselves in the fact that they can annihilate each other in collisions (providing, of course, that there is a third particle which can take up the energy and momentum). It may be maintained (although it does not follow directly from the law of conservation of momentum) that the excitation quanta in this type of spectrum have half-integral momentum and thus obey Fermi statistics. An example of this type is the energy spectrum of the electronic fluid in a nonsuperconducting metal.

We use the symbol $\varepsilon(p)$ to denote the energy of the elementary excitation in a quantum liquid as a function of its momentum p. The manner in which ε depends on p is one of the basic characteristics of a given type of energy spectrum.

In any case, the Fermi type of spectrum cannot result in super-fluidity (cf. below footnote on page 17). Hence it follows that liquid helium is characterized by a Bose spectrum[2].

We consider internal motion in liquid helium, in which the compression waves have wave lengths which are large compared to the interatomic distances. Excitation quanta corresponding to these waves are undoubtedly excited in liquid helium, as in any liquid, and are the so-called acoustic quanta, i.e., phonons; as is well known the phonon energy is a linear function of momentum:

$$\varepsilon = cp \qquad (8.1)$$

(c is the acoustic velocity). Thus, in its initial part (wave lengths associated with small p) the curve which describes the dependence of ε on p should be linear. It is reasonable to assume that the function $\varepsilon(p)$ has no other branch starting from the origin. In other words, all elementary excitations of low energy are phonons. We shall see below that this restriction is sufficient for explaining superfluidity.

We wish to emphasize the following fact. The compression waves, as is well known, are associated with potential (irrotational) motion of the liquid. Thus, it may be said that excited states of helium which are close to the ground state are characterized by pure potential motion.

As the momentum p is increased the curve $\varepsilon = \varepsilon(p)$ starts to exhibit departures from linearity. In general, however, it is impossible to predict the dependence of ε on p for a quantum liquid from theoretical considerations alone; it is necessary to rely on available experimental data such as the measurements of the different thermodynamic properties of helium II (entropy, specific heat) and the velocity of propagation of "second sound" in helium II. An analysis

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[2]We may note that the nature of the energy spectrum of a quantum liquid is not related uniquely to the statistics which govern the particles which make up the liquid. It would be expected, however, that the particle statistics will, to some degree, "favor" a spectrum of the same type. Thus, for example, one would expect that pure He^3 would have a Fermi type spectrum and thus would not be a superfluid.

of these data indicates (Landau [3]) that these quantities can be explained in a reasonable way if it is assumed that the curve $\varepsilon = \varepsilon(p)$ has the form shown in Fig. 1. After the initial linear part (phonons) the energy ε goes through a maximum and then starts to fall off and at some value of the momentum $p = p_0$ the function $\varepsilon(p)$ exhibits a minimum.

At thermal equilibrium the elementary excitations in the liquid are distributed mainly in the energy minima region, that is, in the region of small ε (the region close to $\varepsilon = 0$) and the region close
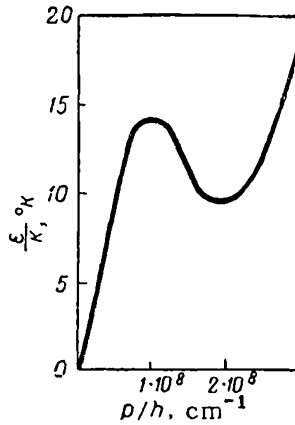
Fig. 1. Energy spectrum of helium II.

to $\varepsilon(p_0)$. Hence these regions are of special importance. Close to the point $p = p_0$ the function $\varepsilon(p)$ may be expanded in a series in powers of $p - p_0$. Since $\dfrac{d\varepsilon}{dp}\bigg|_{p=p_0} = 0$ there is no linear term and including second-order terms we have

$$\varepsilon = \Delta + \frac{(p-p_0)^2}{2\mu},\tag{8.2}$$

where $\Delta = \varepsilon(p_0)$ and μ are constants.

In contrast to phonons, elementary excitations of the second type correspond to vortex motion of the liquid (nonvanishing vortex velocities). These may be called "rotons" and the constant μ in (8.2) may correspondingly be called the "effective mass" of the

roton. It should be kept in mind, however, that in an energy spectrum of the kind described here it is impossible, strictly speaking, to divide the phonons and rotons into two qualitatively different types of elementary excitations since there is a continuous transition from one type to the other. It is more appropriate to speak simply of long-wave (small p) and short-wave (p close to p_0) excitation quanta. Keeping this limitation in mind we will for convenience, however, use the terms phonons and rotons[3].

Knowing the shape of the energy spectrum of the liquid we can calculate the thermodynamic properties. With appropriate choices of the constants Δ, p_0 and μ, the formulas obtained in this manner are in excellent agreement with the available experimental data on the specific heat of helium II (the measurements reported by Keesom; cf. however, footnote on page 14), its entropy (measurements reported by Kapitza) and the velocity of second sound, measured by Peshkov. The constants which determine the energy spectrum of helium II are found to be as follows:

$$\Delta/k = 9.6° \text{ K}, \quad p_0/\hbar = 1.95 \cdot 10^8 \text{ cm}^{-1}, \quad \mu = 0.7 \quad m_{\text{He}}. \tag{8.3}$$

We would like to call attention to the fact that the effective mass of the roton turns out to be of the same order of magnitude as the

[3]In the early version of the theory Landau considered an energy spectrum in which the function $\varepsilon(p)$ had two branches — a phonon branch (8.1) and a roton branch $\varepsilon = \Delta + p^2/2\mu$; both of these started from $p = 0$ so that there was an "energy gap" Δ between the lowest states of the two branches. Aside from the fact that such a spectrum leads to poor agreement with the experimental data it is, in fact, internally inconsistent. The inconsistency results from the fact that the rotons in this spectrum could decay spontaneously into phonons, that is the rotons would be unstable; for example, a roton with energy Δ and momentum $p = 0$ could be converted into two phonons, moving in opposite directions with momenta $p = \Delta/2c$ and energies $\varepsilon = \Delta/2$. We may also note that the energy spectrum of liquid helium has been investigated by Bijl [11] who reached the conclusion that there was a "energy gap" between the ground and the excited states. However, this result must be regarded with some doubt since it means that it would be impossible to have propagation of low-frequency acoustic waves in the liquid.

mass of the helium atom m_{He} while the wave length $\hbar/p_0$ is found to be very small, even smaller than the dimensions of an atom[4].

If the number of rotons and phonons (per unit volume of the liquid) is comparatively small, the system may be considered a mixture of two ideal gases — a phonon gas and a roton gas. This situation obtains at temperatures which are not too close to the λ-point.

In accordance with what has been pointed out above, both the phonon gas and the roton gas obey Bose statistics. However, we may note that since the roton energy always contains the quantity Δ which is large compared with kT (at low temperatures, which are the only temperatures for which we can speak of a roton gas), reasonable accuracy can still be obtained if the Bose distribution is replaced by a Boltzmann distribution.

ENERGY SPECTRUM OF AN ALMOST-IDEAL BOSE-EINSTEIN GAS. As has already been noted the problem of obtaining a complete theoretical picture of the energy spectrum of a real liquid is essentially impossible. In view of this fact, it is desirable to consider any simple model of a macroscopic body, even if it has no direct relation to liquid helium, with the purpose of ascertaining in what way an energy spectrum with characteristics similar to those described above could be produced.

[4]This in no way means that the rotons can be identified with individual helium atoms. We wish to emphasize again that the phonons and rotons are not to associated with any definite atoms or groups of atoms but are merely a means of describing the collective motions of all the atoms of the liquid.

In this connection we may note the following. If we introduce a "roton velocity" $v = d\varepsilon/dp$, from equation (8.2) we find $v = (p-p_0)/\mu$. In particular, at $p = p_0$ it is found that $v = 0$, i.e., when the roton momentum is p_0 the roton has no velocity. This situation in no way introduces internal contradictions and may be explained in terms of the following "classical" analog. Consider a wave packet which propagates in some medium. Its displacement velocity v is determined by the dispersion relation $\omega = \omega(k)$ (ω is the frequency, k is the wave vector) in accordance with the well-known formula for group velocity $v = d\omega/dk$ and may vanish at certain values of k. This means that the wave packet as a whole, i.e., the complete pattern of motion in the medium, remains fixed in space; however, this does not mean that there may not be some kind of internal motion in the medium which may be associated with the transfer of mass in some direction.

The most natural choice for such a model is an "almost-ideal" Bose-Einstein gas, that is, a gas of weakly interacting particles. A consistent quantum-mechanical analysis of this problem was first given by Bogoliubov [12] who, by means of a highly ingenious application of the method of second quantization, was able to determine completely the energy spectrum of a weakly excited state in such a gas.

We refer the reader to the original paper by Bogoliubov for a detailed description of the calculations; here we shall only describe the results. It turns out that a weakly excited state of the gas can be described in terms of a system ("ideal gas") of noninteracting elementary excitations so that the total energy of the gas is made up of the energy of the ground state and the sum of the energies of the individual "quasi-particles" — the elementary excitations. Each elementary excitation may be assigned a momentum p and its energy, as a function of momentum, is given by the expression

$$\varepsilon(p) = \frac{1}{2m} \sqrt{4\rho\nu(p)\,p^2 + p^4},\qquad (8.4)$$

where m is the mass of an individual particle of the gas, ρ is the density, and the function $\nu(p)$ is defined by the integral

$$\nu(p) = \int U(r)\,e^{-ipr/\hbar}\,dV,\qquad (8.5)$$

taken over the entire space, where $U(r)$ is the potential energy associated with the interaction of the two particles of the gas (assumed to depend only on the distance r between particles).

In order for the expression under the radical in (8.4) to be always positive (including small values of the momentum), we require that

$$\nu(0) = \int U(r)\,dV > 0.\qquad (8.6)$$

If this requirement is not observed, at low values of p the energy $\varepsilon(p)$ will be complex, indicating an instability for the excited states of the gas. It can be shown that $\rho\nu(0)/m^2 = \partial P/\partial\rho$ (P is the pressure of the gas), so that the condition in (8.5) is equivalent to the well-known condition for thermodynamic equilibrium of a gas ($\partial P/\partial\rho > 0$). From the mechanical point of view the condition in (8.5) means that the repulsive force predominates in the interaction

between particles. The necessity for introducing this condition into the model being considered is apparent; in the case of a predominant attractive force, at absolute zero and temperatures close to it (corresponding to weakly excited states), the material would not exist as a gas but would condense into a liquid. We may point out that it is in this feature that the model being considered differs from real helium, in which an attractive force operates between atoms.

At small values of the momentum, the energy (8.4) is given by

$$\varepsilon(p) \cong \frac{\sqrt{\rho v(0)}}{m} \, p = \sqrt{\frac{\partial P}{\partial \rho}} \, p = cp \qquad (8.7)$$

$(c = \sqrt{\partial P/\partial \rho}$ is the acoustic velocity in the gas). Thus, in the initial part of the curve $\varepsilon = \varepsilon(p)$ we are, in fact, dealing with phonons. At high values of the momentum, the function $v(p)$ tends toward zero (since it contains a rapidly oscillating exponential factor in the integrand). Thus, at high values of p, we have:

$$\varepsilon(p) \cong \frac{p^2}{2m}, \qquad (8.8)$$

i.e., ε becomes the kinetic energy of the individual particles of the gas.

CALCULATION OF THE THERMODYNAMIC PROPERTIES OF HELIUM II. The features of the energy spectrum of helium II described earlier allow us to draw certain conclusions as to the temperature dependence of the thermodynamic quantities (entropy, specific heat, etc.). The calculations given below refer to temperatures which are not too close to the λ-point so that we can speak of a phonon gas and a roton gas in the liquid. Under these conditions all the thermodynamic quantities are made up of two parts, corresponding to each of these gases respectively; we shall call these the phonon part and the roton part.

The phonon parts of the thermodynamic quantities are determined directly by the formulas of the well-known Debye theory for the specific heat of solid bodies (the thermal excitation is considered a system of phonons). Thus, for the free energy (referred to one gram of the liquid) we have

$$F_{\text{ph}} = -\frac{\pi^2 k^4}{90\hbar^3 c^3 \rho} T^4 \tag{8.9}$$

(ρ is the helium density). The phonon part of the entropy is then obtained by differentiation with respect to temperature

$$S_{\text{ph}} = \frac{2\pi^2 k^4}{45\hbar^3 c^3 \rho} T^3, \tag{8.10}$$

while the phonon part of the specific heat $C_{\text{ph}} = T\dfrac{\partial S_{\text{ph}}}{\partial T}$ is

$$C_{\text{ph}} = \frac{2\pi^2 k^4}{15\hbar^3 c^3 \rho} T^3. \tag{8.11}$$

Thus the phonon part of the specific heat is proportional to the cube of the temperature[5].

In computing the roton parts of the thermodynamic quantities we make use of the fact that at the temperatures being considered the roton gas can be described, with reasonable accuracy, by a Boltzmann distribution. We recall that in this case the number of particles in the roton gas does not remain fixed but is a function of temperature and is determined by the condition of minimum free energy. In accordance with the well-known formula from statistical mechanics, the free energy for an ideal gas with N particles in a volume V is

$$F = -NkT \ln \frac{eV}{N} \int e^{-\varepsilon/kT} \frac{d\tau_p}{(2\pi\hbar)^3}; \qquad d\tau_p = dp_x\, dp_y\, dp_z \; .$$

Setting the derivative $\partial F/\partial N$ equal to zero, we find the number of particles in the roton gas

$$N_{\text{r}} = \frac{1}{\rho\,(2\pi\hbar)^3} \int e^{-\varepsilon/kT} d\tau_{\text{p}}; \tag{8.12}$$

here we have set $V = 1/\rho$, corresponding to the fact that all

[5]These expressions are obtained in the Debye theory by considering single longitudinal acoustic waves and correspond to temperatures which are small compared with the Debye temperature. The last condition is satisfied since the Debye temperature in the present case is

$$\theta = \frac{\hbar c}{k} \left(\frac{18\pi^2 \rho}{m_{\text{He}}} \right)^{1/3} = 30°\text{K}.$$

quantities are referred to unit mass of helium. The corresponding value of the free energy is

$$F_r = -\frac{kT}{\rho(2\pi\hbar)^3}\int e^{-\varepsilon/kT}d\tau_r \, . \tag{8.13}$$

Equation (8.2) for the roton energy must be substituted in these expressions. Since $p_0^2 \gg \mu kT$ in the integration over dp the quantity p in the factor in front of the exponential can, with reasonable accuracy, be replaced by p_0. In the integration of the exponential factor containing the difference $p - p_0$, because of the rapid reduction with increasing $p - p_0$, the limits can be extended from $-\infty$ to $+\infty$. Hence

$$\int p^2 e^{-(p-p_0)^2/2\mu kT}dp \approx p_0^2 \int_{-\infty}^{+\infty} e^{-\xi^2/2\mu kT}d\xi = p_0^2 \sqrt{2\pi\mu kT}.$$

Thus we obtain the following formulas for the number of rotons and the roton part of the free energy:

$$N_r = \frac{2(\mu kT)^{1/2}p_0^2}{(2\pi)^{3/2}\rho\hbar^3}e^{-\Delta/kT}, \tag{8.14}$$

$$F_r = -\frac{2\mu^{1/2}(kT)^{3/2}p_0^2}{(2\pi)^{3/2}\rho\hbar^3}e^{-\Delta/kT}. \tag{8.15}$$

For the roton part of the entropy $S_r = -\dfrac{\partial F_r}{\partial T}$ and specific heat $C_r = T\dfrac{\partial S_r}{\partial T}$ we have

$$S_r = \frac{2(k\mu)^{1/2}p_0^2\Delta}{(2\pi)^{3/2}\rho T^{1/2}\hbar^3}\left(1+\frac{3kT}{2\Delta}\right)e^{-\Delta/kT}, \tag{8.16}$$

$$C_r = \frac{2\mu^{1/2}p_0^2\Delta^2}{(2\pi)^{3/2}\rho k^{1/2}T^{3/2}\hbar^3}\left[1+\frac{kT}{\Delta}+\frac{3}{4}\left(\frac{kT}{\Delta}\right)^2\right]e^{-\Delta/kT} \tag{8.17}$$

Thus the temperature dependence of the roton part of the thermodynamic quantities is basically exponential $(\sim e^{-\Delta/kT})$. At very low temperatures the roton part of the thermodynamic quantities is smaller than the phonon part; at higher temperatures the reverse is true and the roton part becomes greater. Calculations indicate that both parts of the specific heat and entropy become comparable at a temperature of approximately 0.8 - 0.9° K.

In Figures 2 and 3 are shown the entropy and specific heat for helium II computed by these formulas (the calculations have been carried out by Khalatnikov). The values of μ, Δ and p_0 are the same
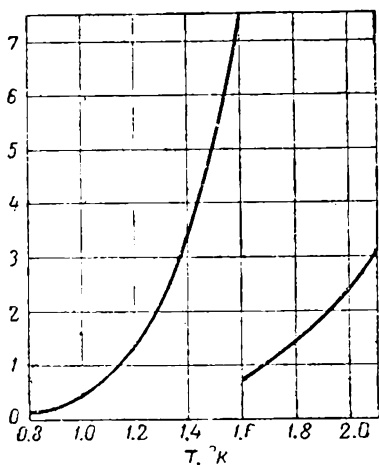
Fig. 2. Entropy of helium ll. Ordinate scale: left-hand curve — 10^{-2} cal/g°; right-hand curve — 10^{-1} cal/g°.
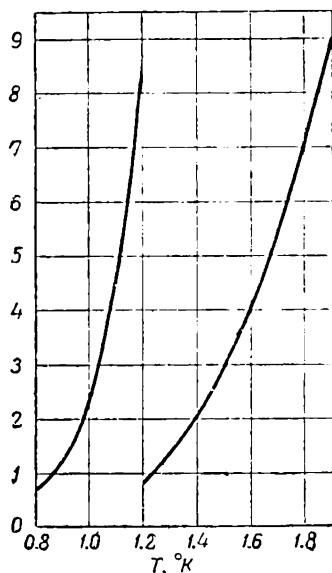
Fig. 3. Specific heat of helium ll. Ordinate scale: left-hand curve — 10^{-2} cal/g°; right-hand curve — 10^{-1} cal/g°.

as in (8.3), the acoustic velocity is taken as 235 m/sec and the helium density as 0.145 g/cm^3. The theoretical formulas may be considerably in error near the λ-point; hence the upper parts of the curve (above 1.8° K) have been extrapolated using the available experimental data[6].

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[6]The following situation should be noted. A careful study of the results of the specific-heat measurements (Keesom and Keesom) and the entropy measurements in helium Il (Kapitza) shows that these data are not consistent. The discrepancy, which leads to a departure from the thermodynamic relation $C = T(\partial S/\partial T)$, indicates the need for a more accurate determination of the parameters which appear in the theoretical formulas.

There is reason to believe that the entropy measurements in the Kapitza experiments are more accurate than the specific-heat measurements carried out by Keesom (first of all, the measurement of specific heat, which involves a measurement of a differential nature is fundamentally a more difficult one; it should also be kept in mind that the temperature intervals in which the specific heat measurements were carried out in the Keesom experiments were relatively large). Hence the comparison between theory and experiment was based specifically on the entropy data for helium II; the curve for the specific heat in Fig. 3 was then computed over the entire temperature region using the theoretical expression (at temperatures above approximately 1.8° K a correction is introduced for the fact that the roton gas is not ideal, cf. Sec. 3).

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Section 2. Superfluidity of Helium II

SUPERFLUIDITY OF HELIUM II AT ABSOLUTE ZERO. We shall now show that the superfluidity of helium II follows from the properties of the energy spectrum described above. We start with an analysis of the liquid helium at absolute zero; at this temperature the liquid is in the ground state.

Consider helium flowing through a capillary with constant velocity $\mathbf{v}$. The existence of viscosity would be manifest in the fact that friction at the walls of the tube and friction inside the liquid itself would cause dissipation of the kinetic energy of the liquid and a gradual reduction of flow.

It will be convenient to consider the flow in a coordinate system which moves with the liquid. In this system the helium is at rest whereas the walls of the capillary move with velocity $-\mathbf{v}$. If there is a viscosity effect, the helium, which is at rest, should be set in motion. It is obvious physically that the entrainment of the liquid by the walls of the tube cannot cause the immediate motion of the liquid as a whole. The motion starts with the excitation of internal motions in the layers of the liquid closest to the wall, i.e., with the appearance of individual excitation quanta in the liquid.

Now suppose that a quantum with momentum $\mathbf{p}$ and energy $\varepsilon(p)$ is excited in the liquid. In this case the energy of the liquid E_0, (in the coordinate system in which it was originally at rest) becomes equal to the energy ε of this quantum and its momentum $\mathbf{p}_0$ is the momentum $\mathbf{p}$ of the quantum. We now return to the coordinate system in which the capillary is at rest. In accordance with the well-known transformation formulas for energy and momentum in classical mechanics, the energy E and the momentum $\mathbf{P}$ of the liquid in this system are

$$E = E_0 + \mathbf{P}_0 \mathbf{v} + \frac{Mv^2}{2}, \qquad \mathbf{P} = \mathbf{P}_0 - M\mathbf{v}$$

(M is the mass of the liquid). Substituting ε and $\mathbf{p}$ in place of E_0 and $\mathbf{P}$ we write

$$E = \varepsilon + \mathbf{p}\mathbf{v} + \frac{1}{2} Mv^2.$$

The term $\frac{1}{2} Mv^2$ represents the original kinetic energy of the moving helium; the expression $\varepsilon + \mathbf{p}\mathbf{v}$, however, is the change in energy due to the excitation associated with the quantum. This

change must be negative inasmuch as the energy of the moving liquid has been reduced:

$$\varepsilon + \mathbf{p}\mathbf{v} < 0.$$

For a given value of p the quantity on the left side of this inequality has a minimum value when $\mathbf{p}$ and $\mathbf{v}$ are antiparallel; hence, in any case we have $\varepsilon - pv < 0$ i.e.,

$$v > \varepsilon/p. \qquad (8.18)$$

This inequality must be satisfied for all values of the momentum of the excitation quantum p. Hence the final condition for the possibility of excitation of a liquid moving in the capillary is obtained by finding the minimum value of ε/p. The expression

$$\frac{d}{dp}\left(\frac{\varepsilon}{p}\right) = \frac{1}{p}\frac{d\varepsilon}{dp} - \frac{\varepsilon}{p^2} = 0,$$

i.e., $\varepsilon/p = d\varepsilon/dp$, determines on the $\varepsilon(p)$ curve, the point at which a line extended from the origin is tangent to the curve. Denoting this value of p by p_m we have, finally,

$$v > \left(\frac{d\varepsilon}{dp}\right)_{p=p_m}. \qquad (8.19)$$

We see that at flow velocities below some definite limit (determined by the slope of the tangent to the curve $\varepsilon = \varepsilon(p)$ drawn from the origin) the helium can not exhibit excitation. This means that the flow of the liquid will not be retarded, i.e., the helium is said to exhibit superfluidity.

At helium velocities which satisfy the condition in (8.19) the superfluidity must vanish in any case. It should be noted, however, that these velocities are extremely large (in order of magnitude $\frac{d\varepsilon}{dp_m} \cong \frac{1}{\mu}\left(\sqrt{2\mu\Delta + p_0^2} - p_0\right) \sim 60\,\text{m/sec}$); these are considerably larger than the so-called "critical velocity" at which superfluidity is actually found to be destroyed. Hence the "critical velocity" must be explained by some other mechanism.

We may note that the important feature of the energy spectrum which leads to the superfluidity property is the fact, in accordance with (8.19), that the derivative on the right-hand side of the in-

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equality is different from zero. This condition leads essentially to the requirement that the term $\varepsilon = \varepsilon(p)$ must not be tangent to the abscissa axis at the origin (neglecting any unlikely tangency to this axis at other points). Hence, superfluidity is a feature of any spectrum in which sufficiently small excitations can be associated with a single phonon[7].

HELIUM II AT TEMPERATURES ABOVE ABSOLUTE ZERO. We now consider helium at temperatures which differ from absolute zero. When helium is not in the ground state, it exhibits certain excitations which, at very low temperatures, may be said to comprise a gas consisting of the excitation quanta. The assumption made above remains valid here since we have not made direct use of the fact that the liquid is originally in the ground state. Thus, again, at temperatures different from 0° K the motion of helium with respect to the walls of the tube cannot lead to the appearance of excitation quanta. On the other hand, it is necessary to take account of quanta in the liquid itself.

For this purpose we consider liquid helium in an axially symmetric container which rotates about its axis with constant angular velocity Ω. We consider a coordinate system which rotates with the container. In this system the container is at rest, that is the liquid is in a state characterized by stationary boundary conditions. Thus, a Gibbs distribution obtains in the liquid, i.e., the probability for finding the liquid in an excited state is given by the expression

$$\text{const.} \cdot e^{-E'/kT},$$

where E' is the energy of the excited state of the liquid in the rotating coordinate system. We now return to the fixed reference system. As is well known, E' the energy of a body in a rotating reference system is related to E the energy in a fixed coordinate system, by the expression

$$E' = E - \mathbf{M}\Omega,$$

where $\mathbf{M}$ is the angular momentum of the body in the given excited

[7]In any case a Fermi type spectrum cannot lead to superfluidity. In a spectrum of this kind it is always possible to have simultaneously two elementary excitations whose total energy is very small but whose momentum is large (a particle and hole near the edge of the Fermi distribution) so that the inequality in (8.18) can be satisfied for any v.

state (in the ground state $\mathbf{M} = 0$). Thus, in the fixed reference system the Gibbs distribution is

$$\text{const.} \cdot e^{-(E-\mathbf{M\Omega})/kT}. \tag{8.20}$$

For convenience it will be assumed that the temperature is low enough so that we can speak of the excited state in terms of an ideal gas of excitation quanta. In this case the energy E and the momentum $\mathbf{M}$ of the excited state can be written in the form

$$E = \sum \varepsilon, \quad \mathbf{M} = \sum \mathbf{m}, \tag{8.21}$$

where ε and $\mathbf{m}$ are the energy and momentum of the individual quanta.

As is well known, substituting the energy in the form $\sum \varepsilon$ in the Gibbs distribution $e^{-E/kT}$ we obtain a distribution for the individual gas "particles" — in the present case a Bose distribution

$$(e^{\varepsilon/kT} - 1)^{-1}$$

(however, the particular distribution is unimportant in what follows).

Similarly, substituting (8.21) in (8.20) we obtain, by the same method, the distribution for the excitation quanta in the rotating container, with the sole difference that instead of ε we have $\varepsilon - \mathbf{\Omega m}$; thus the Bose distribution assumes the form

$$[e^{(\varepsilon - \mathbf{\Omega m})/kT} - 1]^{-1}.$$

But this distribution is nothing more than the distribution to be assigned to a gas which rotates as a whole (with angular velocity Ω). Thus we arrive at the result that statistical equilibrium obtains in a rotating container with helium II and that this equilibrium differs from the equilibrium when the container is at rest only in the fact that the gas of excitation quanta rotates along with the container, as though entrained by the walls.

If now, using the above distribution, we compute the angular momentum of the helium in a rotating container at a given temperature, i.e., the quantity

$$\overline{\mathbf{M}} = \frac{1}{(2\pi\hbar)^3} \int \int \frac{\mathbf{m} \, d\tau_p \, dV}{e^{(\varepsilon - \mathbf{\Omega m})/kT} - 1}, \tag{8.22}$$

at absolute zero, i.e., in the complete absence of excitation quanta,

we find this quantity is obviously zero. At higher temperatures the angular momentum will be different from zero but the moment of inertia (i.e., the proportionality factor between M and Ω) at low temperatures will still be much smaller than the usual moment of inertia (that corresponding to rotation of the entire mass of the liquid with the container).

Thus, we arrive at the fundamental result that in the motion of the walls of the container only part of the mass of the liquid helium is entrained; the other part acts as though it were immobile. For this reason it is often convenient to consider liquid helium as though it were a mixture of two liquids—one, a superfluid, which does not exhibit viscosity and which is not entrained by the walls of the container and the other, a normal fluid, carried along by the motion of the walls and, in general, behaving as a normal fluid. It is important to note that there can be no "friction" as these liquids move "through" each other, that is, there can be no transfer of momentum between them. Actually this mutual motion has been obtained on the basis of statistical equilibrium in a uniformly rotating container. If any relative motion is possible in the state of statistical equilibrium this means that it cannot be accompanied by frictional effects.

We must emphasize that the picture of helium as a "mixture" of two liquids is nothing more than a method of description which is suitable for considering effects which take place in helium II. As is the case when any quantum effect is described in classical terms, such a picture is not completely adequate. Actually, one should say that in a quantum liquid, such as helium II, it is possible to have simultaneously two motions, each of which is associated with its own "effective mass" (such that the sum of both of these masses is equal to the total true mass of liquid). One of these motions is "normal", i.e., it exhibits the properties characteristic of the motion of an ordinary liquid; the other is the "superfluid" motion. Both of these motions take place without the transfer of momentum between themselves. It is especially important to emphasize that we are not making any division of the actual particles of the liquid into "superfluid" and "normal" particles. In a very specific sense we can speak of the superfluid and normal masses of the liquid in terms of masses connected with the simultaneously possible motions; but this in no way implies the possibility of an actual division of the liquid into two parts.

Keeping this limitation with respect to the true nature of the effects in helium II in mind, it is, nonetheless, convenient to use the terms "superfluid" and "normal" as a convenient means for brief descriptions of these effects.

As has been indicated, the superfluid part of the liquid remains immobile as the container with helium is rotated. It may be said that the superfluid is incapable of rotation. Mathematically this situation means that the vortex velocity of the superfluid motion is zero. Thus, the motion of the superfluid is always potential motion. On the other hand, the normal part of the liquid may exhibit both potential and vortex motion.

A particularly interesting property of the motion of helium II follows from the above. As is well known from hydrodynamics, in potential flow a liquid exerts no pressure on streamlined bodies (the so-called Euler paradox). Hence, the superfluid part will exhibit no pressure on a body submerged in helium II; the body will experience a pressure due only to the normal part of the liquid.

An important parameter, which determines the behavior of helium at any given temperature, is the ratio of the mass of the superfluid part to that of the normal part. We introduce the quantities ρ_n the density of the normal liquid, and ρ_s the density of the superfluid; the sum $\rho = \rho_n + \rho_s$ is the total true mass of the liquid.

At absolute zero the ratio ρ_n/ρ is zero. This ratio increases as the temperature is increased, finally reaching a value of unity, at which it remains constant. The temperature at which ρ_n/ρ becomes unity is the point at which helium II becomes helium I. Thus the phase transition in liquid helium is characterized by the disappearance of the superfluid part of the liquid. This disappearance takes place gradually, i.e., ρ_n/ρ becomes unity in a continuous fashion. Thus, this transition is a phase transition of the second kind (there is no evolution or absorption of latent heat). The existence of a discontinuity in the specific heat, however, is, as is well known, a direct thermodynamic characteristic of a phase change of the second kind.

The ratio ρ_n/ρ can be measured directly experimentally by determining the moment of inertia I of a cylindrical container filled with helium II which rotates about its own axis. The ratio

of I to I_0, the moment of inertia computed under the assumption that the entire mass of helium rotates with the container, gives the ratio ρ_n/ρ at a given temperature. Such measurements have been made by Andronikashvili, and the values of ρ_n/ρ have been found to be in good agreement with the theoretical values (as well as with the values which are computed from the velocity of second sound).

The above considerations yield a simple explanation for the experimentally observed properties of helium II. Kapitza has shown that no viscosity is observed in the flow of helium II along a capillary or through a thin slit. From the point of view of the theory presented above this effect is explained by the fact that when helium II flows through a thin slit, the superfluid part flows through the slit without friction; the normal part, however, remains in the container, and flows through the slit relatively slowly with a velocity appropriate to its viscosity and the width of the slit.

A simple explanation is also available for the fact that values different from zero are obtained in measurements of the viscosity of helium II by the damping of tortional oscillations of a disc submerged in the liquid. When the disc rotates in the liquid, which is comprised of a superfluid as well as the normal part, the disc is brought to rest by friction with the normal liquid. Thus, experiments in which one observes the flow along a capillary demonstrate the existence of the superfluid part of the liquid whereas the experiments with the rotating disc in helium II indicate the pressure of the normal part.

The known property of helium II of forming a film which moves along a solid surface is related to its flow along a thin capillary. In itself, the formation of the film is not a property peculiar to helium II. Films are formed by any liquid which wets a solid surface. In ordinary liquids, however, the formation of the film and its spreading over a large portion of the surface takes place extremely slowly because of viscosity effects; in helium II, however, because of the superfluid properties this motion is rapid (Frenkel' [13]; in this work a quantitative description is given of films which form on a vertical wall).

CALCULATION OF THE RATIO ρ_n/ρ . The ratio ρ_n/ρ can be computed at very low temperatures. As has been shown above, the

"ideal gas" of excitation quanta in helium II can be considered as consisting of two parts—a phonon part and a roton part. As a result, at low temperatures ρ_n is made up of two independent parts — the effective mass of the ideal gas of phonons and the mass of the roton gas.

We shall assume that the gas of excitation quanta moves as a whole with a velocity $\mathbf{v}$. As is well known, the distribution function for a gas which moves as a whole is obtained from the distribution function for a gas at rest by simply replacing the particle energy ε by the quantity $\varepsilon - \mathbf{pv}$ where $\mathbf{p}$ is the particle momentum. Thus the total momentum of the gas (referred to unit volume) is

$$\mathbf{P} = \int \mathbf{p} \, n \, (\varepsilon - \mathbf{pv}) \cdot d\tau_{\mathfrak{p}},$$

where $n(\varepsilon)$ is to be taken as a Bose distribution in the phonon case and a Boltzmann distribution in the roton case.

We assume that the velocity $\mathbf{v}$ is small and expand the integrand in powers of $\mathbf{pv}$. The zeroth-order term vanishes in the integration over direction of the vector $\mathbf{p}$, leaving

$$\mathbf{P} = - \int \mathbf{p} \, (\mathbf{pv}) \frac{\partial n \, (\varepsilon)}{\partial \varepsilon} \, d\tau_{\mathfrak{p}}. \tag{8.23}$$

We first consider the phonon gas. In the phonon case $\varepsilon = cp$ so that

$$\frac{c\mathbf{p}}{p} \frac{\partial n}{\partial \varepsilon} = \frac{\partial n}{\partial \mathbf{p}},$$

and we can write

$$\mathbf{P} = - \frac{1}{c} \int p \, (\mathbf{pv}) \frac{\partial n}{\partial \mathbf{p}} \, d\tau_{\mathfrak{p}}.$$

Integrating by parts we have

$$\mathbf{P} = \frac{1}{c} \int \left\{ p\mathbf{v} + \frac{\mathbf{p}}{p} \, (\mathbf{pv}) \right\} n \, d\tau_{\mathfrak{p}}.$$

Now we average the integrand over directions of the vector $\mathbf{p}$. Averaging $\mathbf{p}(\mathbf{pv})$ yields $\frac{1}{3} \, p^2 \, \mathbf{v}$ so that

$$\mathbf{P} = \frac{4}{3c} \, \mathbf{v} \int p n \, d\tau_{\mathfrak{p}} = \frac{4}{3c^2} \, \mathbf{v} \int \varepsilon n \, (\varepsilon) \, d\tau_{\mathfrak{p}}.$$

Now the integral is nothing more than the energy ρE_{ph} per unit volume of the phonon gas, whence

$$\mathbf{P} = \frac{4E_{ph}}{3c^2}\rho\mathbf{v}.$$

The coefficient which multiplies $\mathbf{v}$ is the effective mass per unit volume of the phonon gas, i.e., the phonon part $(\rho_n)_{ph}$ of the density ρ_n :

$$(\rho_n)_{ph} = \frac{4E_{ph}}{3c^2}\rho = \frac{2\pi^2 k^4}{45\hbar^3 c^5}T^4 \tag{8.24}$$

[for the energy $E_{ph} = F_{ph} + TS_{ph}$ we use equations (8.9) and (8.10)]. Thus the phonon part of ρ_n is proportional to T^4.

In computing the roton part of $\mathbf{v}$ we may note that in the Boltzmann distribution

$$\frac{\partial n(\varepsilon)}{\partial \varepsilon} = -\frac{n}{kT} ;$$

averaging the integrand in equation (8.23) over directions of $\mathbf{p}$ we have

$$\mathbf{P} = \frac{\mathbf{v}}{3kT}\int p^2 n(\varepsilon)\, d\tau_p = \mathbf{v}\frac{\overline{p^2}}{3kT}\rho N_r,$$

where $\overline{p^2}$ is the mean value of the square of the roton momentum.

Thus we arrive at the formula

$$(\rho_n)_r = \frac{\overline{p^2}}{3kT}\rho N_r. \tag{8.25}$$

Since $p_0^2 \gg \mu kT$ we can write $\overline{p^2} = p_0^2$ and obtain

$$(\rho_n)_r = \frac{p_0^2}{3kT}\rho N_r = \frac{2\mu^{1/2}p_0^4}{3(2\pi)^{3/2}(kT)^{1/2}\hbar^3}e^{-\Delta/kT}. \tag{8.26}$$

[N_r is obtained from equation (8.14)]. Thus the roton part of ρ_n is an exponential function of temperature.

The phonon part of ρ_n becomes important only at extremely low temperatures. Even at $1°$ K the phonon part is 60 times smaller than the roton part; the two parts become comparable at approximately $0.6°$ K.

It is apparent that the curve $\rho_n(T)$ cannot be computed exactly near the λ-point. However, because of the very rapid increase in ρ_n using equation (8.26) the temperature of the λ-point can be obtained approximately by taking $\rho_n/\rho = 1$ and using equation (8.26). This calculation gives a value of 2.38° K for the λ-point, a value which is in good agreement with the actual value of 2.19° K.

In Figure 4 is shown the temperature dependence of ρ_n/ρ obtained from the above formula (the calculation has been carried out by Khalatnikov). The upper part of the curve is obtained by interpolation of the experimental data.

HEAT TRANSPORT IN HELIUM II. The entropy of helium II is determined by the statistical distribution of the excitation quanta. Thus, in any motion of the liquid for which the gas of excitation quanta remains fixed, there will be no macroscopic transfer of entropy. We thus arrive at the very important result that there is no transfer of entropy in the flow of a superfluid. In other words, the superfluid does not transport heat in its motion. Whence, in turn, it follows that motion of helium II in which only the superfluid takes part is thermodynamically reversible.

As has already been indicated, the flow of helium II through a thin capillary or slit is the flow of a superfluid. Hence, such a flow must be reversible (more accurately, complete reversibility is approached as the capillary becomes narrower and the flow of the normal part becomes smaller). This situation has actually been observed in the experiments performed by Kapitza.

Moreover, since the helium which flows through a thin capillary carries no heat, we can draw a further conclusion: liquid helium which flows from a container through a thin capillary should be at a temperature of absolute zero (more accurately, at a temperature lower than the temperature of the helium in the container and zero only in the idealized case of an infinitesimally thin capillary).

The mechanism for heat transport in helium II is the transfer of heat by virtue of the movement of the normal part. It is of a peculiar convective nature and differs in principle from ordinary heat conductivity. Any difference of temperature in helium II leads rapidly to internal motion of the normal and superfluid parts
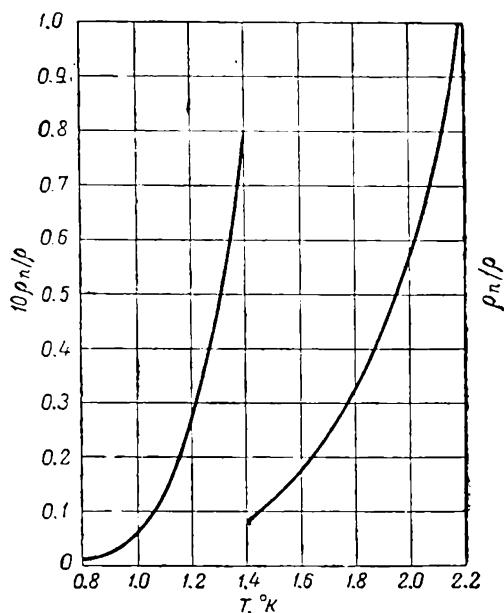
Fig. 4. Temperature dependence of the ratio ρ_n/ρ.

"through" each other; there need not be any actual macroscopic flow characterized by the transfer of mass in the fluid.

This is the case in heat transport in a capillary, the large magnitude of which was discovered by Keesom and Keesom and which was studied in detail by Kapitza. When the helium at one end of the capillary is heated two oppositely directed flows are produced. The normal fluid moves from the heated end to the cold end, carrying heat; the heat carried in this way easily serves to explain the experimentally observed high heat transfer. The superfluid flows in the opposite direction. Both flows compensate each other exactly as far as the transfer of mass is concerned so that there is no actual macroscopic flow in the helium.

Kapitza observed the deflection of a thin vane which was suspended in front of the open end of the capillary when the helium at the closed end was heated. This effect can be explained as follows: the superfluid flowing into the capillary exerts no pressure on the streamlined vane (because of the potential motion). On the other hand, the normal liquid flowing from the capillary does exert a pressure on the vane, deflecting it away from the end of the capillary. We may note the following very unique situation — it is possible to have a force operating on a body which is suspended in helium II even though the liquid as a whole is at rest.

MECHANO-CALORIC EFFECT IN HELIUM II. As is well known, the so-called mechano-caloric effect in helium II refers to the fact that when helium flows from a container through a thin capillary cooling of the container is observed. Thermo-mechanical effects are not peculiar to helium; the anomaly in helium II is the large magnitude of this effect. In ordinary liquids thermo-mechanical effects are irreversible (such as the Peltier thermal-electric effect).

The latter type of effect should be observed in helium II, but it is masked by stronger effects which are peculiar to helium II and which have nothing in common with irreversible effects such as the Peltier effect. In particular, the heating which is observed on outflow is simply a result of the fact that the helium which flows through the capillary carries no heat; thus the heat available in the container is distributed over a smaller amount of helium II. When helium flows into the container the opposite holds true.

It is easy to determine the amount of heat Q absorbed when one gram of helium flows into the container through the capillary. Since the inflowing liquid does not contain excitation quanta its entropy is zero. In order that the helium in the container remain at a temperature T it is necessary to impart to it an amount of heat TS (S is the entropy of one gram of helium at temperature T) to compensate for the reduction of the entropy per unit mass by virtue of the introduction of one gram of helium with zero entropy. This means that when one gram of helium at temperature T flows into the container there is absorbed an amount of heat

$$Q = TS \tag{8.27}$$

Conversely, when one gram of helium at temperature T flows out of the container an amount of heat TS is generated.

Now consider two containers with helium II at temperatures T_1 and T_2, respectively; the containers are connected by a thin capillary. Because of the possibility of free flow of the superfluid in the capillary, mechanical equilibrium of the helium in both containers is established rapidly. However, inasmuch as the superfluid does not transfer heat, thermal equilibrium (for which the temperatures of the helium in both containers become equilibrated) is established at a much slower rate.

The condition for mechanical equilibrium of the helium is easily written making use of the fact that this equilibrium, in accordance with the foregoing, takes place at constant entropy S_1 and S_2 of the helium in both containers. If E_1 and E_2 are the internal energies per unit mass of helium at temperatures T_1 and T_2 the condition of mechanical equilibrium (i.e., minimum energy) realized by the transfer of the superfluid is

$$\left(\frac{\partial E_1}{\partial N}\right)_{S_1} = \left(\frac{\partial E_2}{\partial N}\right)_{S_2},$$

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where N is the number of atoms in one gram of helium. But the derivative $(\partial E/\partial N)_S$ is the chemical potential ζ. Hence we have the equilibrium condition in the form

$$\zeta(P_1, T_1) = \zeta(P_2, T_2)$$

(P_1, P_2 are the pressures in the first and second containers) or

$$\Phi(P_1, T_1) = \Phi(P_2, T_2),$$

where Φ is the thermodynamic potential of helium II.

If the pressures P_1 and P_2 are small, using a power expansion and noting that $\partial\Phi/\partial P$ is divided by the volume V we find

$$V\Delta P = \Phi(T_1) - \Phi(T_2) = \int_{T_1}^{T_2} S dT$$

($\Delta P = P_2 - P_1$). If the temperature difference $\Delta T = T_2 - T_1$, is also small, expanding in powers of ΔT and noting that $\partial\Phi/\partial T = -S$ we find

$$\frac{\Delta P}{\Delta T} = \frac{S}{V}. \tag{8.28}$$

Since $S > 0$, $V > 0$ then $\Delta P/\Delta T > 0$ in accordance with experiment. Equations (8.27) and (8.28) have been completely verified by the experiments performed by Kapitza.

These formulas have been obtained by London [14], starting from the analysis given by Tisza, which is in qualitative agreement with the Landau theory on this point.

It is interesting to note that these effects can be considered a sort of osmosis in a "solution" of excitation quanta in liquid helium, in which the thin capillary or slit plays the role of the semi-permeable membrane.

EFFECT OF IMPURITY ATOMS IN HELIUM II. It is of interest to examine the behavior of impurities in helium II; in particular, it is of interest to examine the effect of the helium isotope (He^3), mixed, in very small concentrations, with the main isotope (He^4); the analysis given follows that of Landau and Pomeranchuk [15].

To analyze this problem it is first necessary to consider the energy spectrum of the system consisting of helium II plus the impurity atoms (the impurity concentration is assumed to be so small that the impurity atoms do not interact with each other). The presence of an impurity atom in the liquid leads to the appearance of a new branch in the energy spectrum, corresponding to the motion of this atom through the liquid; in view of the strong interaction between the impurity atom and the atoms of the liquid it is reasonable to assume that the motion will actually be a "collective" effect, in which the helium atoms also take part. It is possible to assign to this motion some resultant momentum, p', which is conserved.

Thus, elementary excitations of a new type appear in the liquid (the number is equal to the number of impurity atoms); the energy ε' of these excitations is a definite function of the momentum p'. At statistical equilibrium these excitations will have energy values which are concentrated about the minima of the function $\varepsilon'(p')$. Close to the minimum the function $\varepsilon'(p')$ is of the form

$$\varepsilon' = (p' - p_0)^2 / 2\mu' \tag{8.29}$$

(ε' is computed from the minimum value), where p_0 may be zero in particular cases and μ' is the effective mass associated with the motion of the impurity atom[8].

These new "elementary excitations" interact with the phonons and rotons, colliding with them and thus entering into the makeup of the "normal part" of the liquid. Thus, for example, in the passage of helium II through a thin slit the impurity atoms are filtered along with all the "normal liquid"[9]. We wish to emphasize that this circumstance has no bearing on whether or not the impurity itself (in particular, pure He^3 isotope) exhibits the superfluid property, in spite of the erroneous statements on this point which are found in the literature [10], [16].

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[8]The statistics which govern these new elementary excitations are the same as the statistics for the impurity atoms.

[9]This "filtering" of the He^3 isotope has actually been observed by Daunt, Nier et al.

Finally, the number of impurity atoms remains unchanged as the temperature is reduced whereas the number of rotons and phonons falls off rapidly. Hence, at sufficiently low temperatures the contribution of the impurities to ρ_n and to the thermodynamic quantities in helium II becomes predominant. Thus, for the "impurity part" of the density ρ_n, in analogy with (8.26) we obtain (with $p'_0 \neq 0$).

$$(\rho n)_{\text{admix}} = \frac{p'^2_0}{3kT} \rho N_{\text{admix}}$$

where N_{admix} is the number of impurity atoms in one gram of helium (it is assumed that Boltzmann statistics apply to the "impurity excitation quanta"). If, however, $p_0 = 0$ then in the same way [using equation (8.24)] we obtain the result

$$(\rho n)_{\text{admix}} = \mu' N_{\text{admix}}$$

Thus, $(\rho n)_{\text{admix}}$ remains constant or may even increase as the temperature is reduced.

Section 3. Macroscopic Hydrodynamics of Helium II

SYSTEM OF HYDRODYNAMIC EQUATIONS FOR HELIUM II. Starting from the ideas developed above concerning the microscopic mechanism for superfluidity, it is possible to construct a complete system of hydrodynamic equations which can describe helium II in macroscopic fashion (phenomenological theory).

The starting point is the fundamental postulate that in the quantum liquid — helium II — there take place simultaneously two motions and, correspondingly, that the motion of helium II must be described at each point using two velocities rather than one, as in ordinary hydrodynamics. We use the symbols v_s and v_n to denote the velocities associated with the "superfluid" and "normal" motions. Since the superfluid motion is always potential, we have

$$\text{curl } v_s = 0. \tag{8.30}$$

It turns out that the hydrodynamic equations with two velocities v_s and v_n can be obtained in completely unique fashion from the sole requirement that these equations satisfy all the conservation laws. For the general case of arbitrary velocities these equations become extremely complicated and shall not be treated here; rather

we limit ourselves to a simplified derivation of the equations which apply to motion for which the velocities v_s and v_n are not too large.

We use the symbol $\mathbf{j}$ to denote the macroscopic mass flow of the liquid. This quantity is function of both velocities v_s and v_n and, at small velocities, can be expanded in a power series. In the first approximation

$$\mathbf{j} = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n. \tag{8.31}$$

The coefficients ρs and ρn are obviously the quantities which we have called the densities of the superfluid and normal parts of the liquid. The sum of these two coefficients is the density ρ of the helium II:

$$\rho = \rho_s + \rho_n \tag{8.32}$$

Finally, ρs and ρn are functions of the temperature. We may note that the mass flow $\mathbf{j}$ is nothing more than the momentum density, i.e., the momentum per unit volume of the liquid.

The density ρ and the flow $\mathbf{j}$ must satisfy the equation of continuity:

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \mathbf{j} = 0. \tag{8.33}$$

For the time being we consider only motion of the liquid for which the viscosity of the normal part can be neglected. In this case the equation for the conservation of momentum is written in the form

$$\frac{\partial j_i}{\partial t} + \sum_{k=1}^{3} \frac{\partial \Pi_{ik}}{\partial x_k} = 0, \tag{8.34}$$

where Π_{ik} is the momentum flow density tensor:

$$\Pi_{ik} = P\delta_{ik} + \rho_s v_{si} v_{sk} + \rho_n v_{ni} v_{nk}, \tag{8.35}$$

where P is the pressure (we may recall that in ordinary hydrodynamics $\Pi_{ik} = P\delta_{ik} + \rho v_i v_k$).

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In the absence of any viscosity effects for the normal part of the liquid the motion of helium II is reversible. Thus this motion must satisfy the equation for the conservation of entropy:

$$\frac{\partial (\rho S)}{\partial t} + \mathrm{div}_i'(\rho S \mathbf{v}_n) = 0 \qquad (8.36)$$

(S is the entropy per unit mass of helium II). The "entropy flow" is $\rho S \mathbf{v}_n$ since entropy is transported only by the normal part of the liquid. Correspondingly the heat flow $\mathbf{q}$ is

$$\mathbf{q} = \rho T S \mathbf{v}_n. \qquad (8.37)$$

Finally, the last equation for the complete system of hydrodynamic equations is obtained by equating the acceleration $d\mathbf{v}_s/dt$ and the force which acts on a unit superfluid mass. To determine this force we imagine that a unit mass of the liquid is transported from point "1" to point "2" in such a way that the distribution of excitation quanta in the liquid is not changed. In other words, it may be said that in the transfer only the superfluid liquid is displaced while the distribution of the normal liquid remains unchanged. In this transfer the energy E of the liquid is changed by

$$\left(\frac{\partial E}{\partial M}\right)_1 - \left(\frac{\partial E}{\partial M}\right)_2$$

(M is the mass of the liquid). The derivatives here are to be taken at constant entropy (since the entropy is associated only with the normal liquid) and for constant momentum of the motion of the normal mass of liquid with respect to the superfluid mass[10]; furthermore, it is assumed that the volume of the liquid is constant.

From the expression which has been written for the change in energy it is apparent that the quantity $\partial E/\partial M$ may be considered

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[10]The motion of the superfluid determines the external conditions in which the excitation quanta move. Hence, the Lagrangian function for the motion of the normal part of the liquid does not depend simply on its velocity $\mathbf{v}_n$, but rather on the velocity difference $\mathbf{v}_n - \mathbf{v}_s$. The momentum being conserved thus appears as a derivative of the Lagrangian with respect to $\mathbf{v}_n - \mathbf{v}_s$, i.e., the momentum of the relative motion.

a "potential energy" for the superfluid so that the force acting on it is

$$- \operatorname{grad} \frac{\partial E}{\partial M} \,.$$

To compute the derivative $\partial E/\partial M$ we note that the derivative of energy at constant entropy and volume is equal to the derivative of the thermodynamic potential at constant temperature and pressure. The thermodynamic potential of the liquid $M \Phi$ (Φ is the potential per unit mass) may be written as a sum of the thermodynamic potentials for the immobile liquid and the kinetic energy $\mathbf{P}^2/2\,Mn$ of the relative motion of the superfluid and normal parts:

$$M\Phi = M\Phi_0 + \frac{\mathbf{P}^2}{2M_n} \,.$$

Here $\mathbf{P}$ is the momentum associated with the motion of the normal mass with the respect to the superfluid. Differentiating $M\Phi$ with respect to M at constant $P, T, \mathbf{P}$ and assuming that the normal mass Mn is proportional (for a given P and T) to the total mass M, we have

$$\Phi_0 - \mathbf{P}^2/2M_n M.$$

Substituting everywhere $\mathbf{P} = Mn\,(\mathbf{v}_n - \mathbf{v}_s)$ and replacing the ratio of the masses by the ratio of the densities we arrive at the derivative being sought $(\partial E/\partial M)_{S, V, \mathbf{P}}$

$$\Phi_0 - \frac{\rho_n}{2\rho}(\mathbf{v}_n - \mathbf{v}_s)^2.$$

Consequently the hydrodynamic equation is of the form

$$\frac{d\mathbf{v}_s}{dt} \equiv \frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{v}_s \nabla)\,\mathbf{v}_s = - \operatorname{grad}\left\{\Phi - \frac{\rho_n}{2\rho}(\mathbf{v}_n - \mathbf{v}_s)^2\right\}$$

(the subscript on Φ_0 is omitted). This can be written in another way, noting that since $\operatorname{curl}\,\mathbf{v}_s = 0$ everywhere,

$$(\mathbf{v}_s \nabla)\,\mathbf{v}_s = \operatorname{grad}\frac{v_s^2}{2}\,.$$

Thus

$$\frac{\partial \mathbf{v}_s}{\partial t} = - \operatorname{grad}\left\{\Phi + \frac{v_s^2}{2} - \frac{\rho_n}{2\rho}(\mathbf{v}_n - \mathbf{v}_s)^2\right\}\,. \tag{8.38}$$

Equations (8.30) - (8.38) constitute a complete set of hydrodynamic equations for helium II. It remains only to write the

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boundary conditions for these equations. First of all, at any solid (fixed) surface the component of the mass flow j perpendicular to this surface vanishes. Further, it must always be understood that the "normal part" of the liquid is actually an ensemble of thermal excitations. In moving along a solid surface the excitation quanta interact with the surface and this is described macroscopically as the "sticking" of the normal part of the liquid to the wall, as in ordinary viscous liquids. Thus, at a solid surface the tangential component of the velocity v_n must vanish.

As far as the component of v_n perpendicular to the wall is concerned, it must be kept in mind that the excitation quanta can be absorbed by a solid body — this corresponds simply to heat transfer from the liquid to the solid body. Hence, the velocity component of v_n perpendicular to the wall cannot become zero in continuous fashion; the boundary conditions require only the continuity of q the heat flux perpendicular to the wall (8.37). Finally, the temperature must be continuous.

Thus the boundary conditions at the surface of a fixed solid body can be written in the following form (we employ a system of coordinates in which the x-axis is perpendicular to the surface at the point in question):

$$\rho_s v_{sx} + \rho_n v_{nx} = 0, \qquad v_{ny} = v_{nz} = 0, \tag{8.39}$$

$$\rho T S v_{nx} = -\varkappa \left(\frac{\partial T}{\partial x}\right)_{\text{sol}}, \qquad T = T_{\text{sol}}, \tag{8.40}$$

where $\varkappa$ is the thermal conductivity for the solid body.

Actually, however, down to the lowest possible temperatures the heat transfer in the solid body is extremely small as compared with the heat transfer in helium II and effects due to the thermal conductivity of the solid body are negligible. Under these conditions we can set the quantity $\varkappa$ equal to zero and the boundary conditions then assume the form

$$v_{sx} = 0, \ v_n = 0. \tag{8.41}$$

In other words, we are dealing with the usual boundary conditions for an ideal liquid for v_s and a viscous liquid for v_n.

HYDRODYNAMIC EQUATIONS FOR AN INCOMPRESSIBLE LIQUID.

We consider motion of helium II for which it may be considered an incompressible liquid, as in the usual case for viscous flow (Landau [2]). We also take into account the viscosity of the normal part of the liquid.

In order to take account of the viscosity of the normal part of the liquid, in equation (8.34) it is necessary to add terms to the tensor Π_{ik}; these are expressed in the usual way through the coefficient of viscosity and the derivatives of the velocity V_n with respect to coordinates. For an incompressible liquid

$$\Pi_{ik} = P\delta_{ik} + \rho_s v_{si} v_{sk} + \rho_n v_{ni} v_{nk} - \eta \left(\frac{\partial v_{ni}}{\partial x_k} + \frac{\partial v_{nk}}{\partial x_i} \right), \qquad (8.42)$$

where η is the coefficient of viscosity for the normal liquid[11]. Taking account of viscosity also leads to the appearance of additional terms on the right-hand side of equation (8.36), which express the increase in entropy as a result of the irreversible processes of viscous flow. In general, however, these terms are of higher order than the additional terms in equation (8.34) and may be neglected as before.

Assuming that the densities ρs and ρn and the entropy S are constant, from equation (8.36) we have $\mathrm{div}\, V_n$ = 0 while from equation (8.33) div $\mathbf{j}$ = 0, or

$$\mathrm{div}\, \mathbf{v}_n = \mathrm{div}\, \mathbf{v}_s = 0. \qquad (8.43)$$

Keeping these relations in mind and substituting (8.42) in (8.34) we obtain the equation

$$\rho_s \frac{\partial \mathbf{v}_s}{\partial t} + \rho_n \frac{\partial \mathbf{v}_n}{\partial t} + \rho_s (\mathbf{v}_s \nabla) \mathbf{v}_s + \rho_n (\mathbf{v}_n \nabla) \mathbf{v}_n = - \nabla P + \eta \Delta \mathbf{v}_n. \qquad (8.44)$$

Equation (8.38), however, remains unchanged.

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[11]In a compressible liquid, in Π_{ik} there is still one other term which is proportional to div $\mathbf{v}_n$ and for which the "second viscosity" is the coefficient. Moreover, in a compressible liquid additional terms proportional to div $\mathbf{v}_n$ should be added in equation (8.38) (in the expression for the grad operator). Thus, in the hydrodynamics of helium II, generally speaking, there should be not one but several different "second viscosity" coefficients.

Inasmuch as the superfluid motion is potential, we may introduce the potential φ_s associated with the velocity $\mathbf{v}_s$ using the relation

$$\mathbf{v}_s = \operatorname{grad} \varphi_s. \tag{8.45}$$

Since div $\mathbf{v}_s = 0$, the potential φ_s must satisfy Laplace's equation

$$\Delta \varphi_s = 0. \tag{8.46}$$

Introducing φ_s in equation (8.44) and writing   we have

$$\rho_n \frac{\partial \mathbf{v}_n}{\partial t} + \rho_s \frac{\partial \mathbf{v}_s}{\partial t} + \rho_n (\mathbf{v}_n \nabla) \mathbf{v}_n + \rho_s \operatorname{grad} \frac{v_s^2}{2} +$$
$$+ \rho_s \operatorname{grad} \frac{\partial \varphi}{\partial t} = -\nabla P + \eta \Delta \mathbf{v}_n.$$

We introduce two additional quantities. The "pressures" of the normal and superfluid flow P_n and P_s for which the following equation is satisfied:

$$P = P_0 + P_n + P_s, \tag{8.47}$$

where P_0 is the pressure at infinity while P_s is defined by the usual formula for an ideal liquid:

$$P_s = -\rho_s \frac{\partial \varphi_s}{\partial t} - \frac{\rho_s v_s^2}{2}. \tag{8.48}$$

The equation of motion for the velocity $\mathbf{v}_n$ then assumes the form

$$\frac{\partial \mathbf{v}_n}{\partial t} + (\mathbf{v}_n \nabla) \mathbf{v}_n = -\frac{1}{\rho_n} \nabla P_n + \frac{\eta}{\rho_n} \Delta \mathbf{v}_n. \tag{8.49}$$

Formally, this equation is identical with the Navier-Stokes equation for a liquid of density ρn and viscosity η (and correspondingly, kinematic viscosity $\eta / \rho n$).

In other words, the problem of the motion of incompressible helium II is reduced to two problems in ordinary hydrodynamics, one for an ideal liquid and one for a viscous liquid. Specifically, the distribution of the superfluid velocity $\mathbf{v}_s$ is determined by the solution of Laplace's equation (8.46) with the boundary conditions for $\partial \varphi_s / \partial n$ the same as in the usual problem of the potential flow

of an ideal liquid. Furthermore, the distribution of the normal velocity v_n is determined by the solution of the Navier-Stokes equation (8.49) with the same boundary conditions on v_n as in the usual problem for the flow of a viscous liquid. The pressure distribution is then determined from (8.47).

Finally, we derive an expression for the temperature distribution in moving helium II.

Writing $v_s = \nabla \varphi_s$ in equation (8.38) and integrating, we have

$$\Phi + \frac{v_s^2}{2} - \frac{\rho_n}{2\rho} (\mathbf{v}_n - \mathbf{v}_s)^2 + \frac{\partial \varphi_s}{\partial t} = \text{const.} \tag{8.50}$$

The variations in temperature and pressure in an incompressible liquid are small and thus the thermodynamic potential can be expanded in powers of $T - T_0$ and $P - P_0$ (T_0 and P_0 are the temperature and pressure at infinity). Including second-order terms we have

$$\Phi - \Phi_0 = -S(T - T_0) + \frac{1}{\rho} (P - P_0).$$

Substituting this expression in equation (8.50) we have

$$-S(T - T_0) + \frac{1}{\rho} (P - P_0) + \frac{v_s^2}{2} - \frac{\rho_n}{2\rho} (\mathbf{v}_n - \mathbf{v}_s)^2 + \frac{\partial \varphi_s}{\partial t} = 0.$$

Finally, introducing P_s and P_n we have

$$T - T_0 = \frac{\rho_n}{\rho S} \left\{ \frac{P_n}{\rho_n} - \frac{P_s}{\rho_s} - \frac{(\mathbf{v}_n - \mathbf{v}_s)^2}{2} \right\}. \tag{8.51}$$

PROPAGATION OF SOUND IN HELIUM II. We now apply the hydrodynamic equations for helium II to the propagation of sound in this liquid. Usually, for a sound wave the velocities are assumed small while the density, pressure, and entropy are essentially equal to their fixed equilibrium values. Under these conditions the system of hydrodynamic equations can be linearized — in equations (8.35) and (8.38) we neglect quadratic terms in velocity while in equation (8.36) in the term div $(S\rho v_n)$ the entropy ρS can be taken out from under the div sign (since this term contains the quantity v_n which is assumed small). Under these assumptions the system of hydrodynamic equations assumes the form

$$\frac{\partial \rho}{\partial t} + \text{div } \mathbf{j} = 0, \tag{8.52}$$

$$\frac{\partial (S\rho)}{\partial t} + S\rho \ \text{div} \ \mathbf{v}_n = 0, \tag{8.53}$$

$$\frac{\partial \mathbf{j}}{\partial t} + \nabla P = 0, \tag{8.54}$$

$$\frac{\partial \mathbf{v}_s}{\partial t} + \nabla \Phi = 0. \tag{8.55}$$

Differentiating (8.52) with respect to time and substituting in (8.54), we have

$$\frac{\partial^2 \rho}{\partial t^2} = \Delta P. \tag{8.56}$$

The following relation applies for the thermodynamic potential:

$$d\Phi = -SdT + VdP = -SdT + \frac{1}{\rho} \, dP$$

(V is the specific volume). Whence

$$\nabla P = \rho S \nabla T + \rho \nabla \Phi$$

or, substituting ∇P from equation (8.54) and $\nabla \Phi$ from equation (8.55),

$$\rho_n \frac{\partial}{\partial t} (\mathbf{v}_n - \mathbf{v}_s) + \rho S \Delta T = 0.$$

We now apply the div operator to this equation, substituting for div $(\mathbf{v}_s - \mathbf{v}_n)$ the expression

$$\text{div} \, (\mathbf{v}_s - \mathbf{v}_n) = \frac{\rho}{\rho_s S} \frac{\partial S}{\partial t} \, ,$$

which follows from the relation

$$\frac{\partial S}{\partial t} = \frac{1}{\rho} \frac{\partial (S\rho)}{\partial t} - \frac{S}{\rho} \frac{\partial \rho}{\partial t} = -S \, \text{div} \, \mathbf{v}_n + \frac{S}{\rho} \, \text{div} \, \mathbf{j} =$$

$$= \frac{S\rho_s}{\rho} \, \text{div} \, (\mathbf{v}_s - \mathbf{v}_n).$$

As a result we obtain the equation

$$\frac{\partial^2 S}{\partial t^2} = \frac{\rho_s S^2}{\rho_n} \Delta T. \tag{8.57}$$

Equations (8.56) and (8.57) determine the propagation of sound in helium II. From the very fact that there are two equations it is

obvious that in helium II there must exist two propagation velocities.

We write S, P and T in the form $S = S_0 + S'$, $P = P_0 + P'$ etc., where the primed quantities denote the small changes in the corresponding quantities due to the sound wave, while the quantities with the subscript "0" represent the equilibrium values of these same quantities. Thus,

$$\rho' = \frac{\partial \rho}{\partial P} P' + \frac{\partial \rho}{\partial T} T', \quad S' = \frac{\partial S}{\partial P} P' + \frac{\partial S}{\partial T} T',$$

and equations (8.56) and (8.57) assume the form

$$\frac{\partial \rho}{\partial P} \frac{\partial^2 P'}{\partial t^2} - \Delta P' + \frac{\partial \rho}{\partial T} \frac{\partial^2 T'}{\partial t^2} = 0,$$

$$\frac{\partial S}{\partial P} \frac{\partial^2 P'}{\partial t^2} + \frac{\partial S}{\partial T} \frac{\partial^2 T'}{\partial t^2} - \frac{S^2 \rho_s}{\rho_n} \Delta T' = 0.$$

We take the solution of these equations in the form of a plane wave in which P' and T' are proportional to the factor $e^{i\omega(t-x/u)}$ (u is the sound velocity). The condition of compatibility for both equations is the equation

$$u^4 \frac{\partial(S, \rho)}{\partial(T, P)} - u^2 \left(\frac{\partial S}{\partial T} + S^2 \frac{\rho_s}{\rho_n} \frac{\partial \rho}{\partial P} \right) + \frac{\rho_s S^2}{\rho_n} = 0$$

(where

$$\frac{\partial(S, \rho)}{\partial(T, P)}$$

denotes the Jacobian of the transformation of S, ρ with respect to T and P).

Using a simple transformation based on well-known thermodynamic relations this equation may be written in the form

$$u^4 - u^2 \left[\left(\frac{\partial P}{\partial \rho} \right)_S + \frac{\rho_s T S^2}{\rho_n C_v} \right] + \frac{\rho_s T S^2}{\rho_n C_v} \left(\frac{\partial P}{\partial \rho} \right)_T = 0 \qquad (8.58)$$

(C_v is the specific heat per unit mass). This quadratic equation (in u^2) determines the two propagation velocities of sound in helium II.

When $\rho s = 0$, that is, at the λ-point, one of the roots of equation (8.58) vanishes and, as is to be expected, we obtain only the single usual velocity of sound $u = \sqrt{(\partial P/\partial \rho)_S}$ (which we have denoted above by the symbol c).

Actually, at all temperatures the specific heats C_p and C_v are essentially the same. In accordance with well-known thermodynamic relations, under these conditions the isothermal and adiabatic compressibilities are also approximately the same:

$$\left(\frac{\partial P}{\partial \rho}\right)_T \cong \left(\frac{\partial P}{\partial \rho}\right)_S .$$

Denoting the common value of C_p and C_v by C and the common values $(\partial P/\partial \rho)_S$ and $(\partial P/\partial \rho)_T$ by $\partial P/\partial \rho$, from equation (8.58) we obtain the following two expressions for the velocities of sound:

$$u_1 = \sqrt{\frac{\partial P}{\partial \rho}}, \qquad u_2 = \sqrt{\frac{TS^2 \rho_s}{C \rho_n}}. \tag{8.59}$$

Thus, one of the velocities (u_1) is essentially constant while the other (u_2) is a strong function of temperature, becoming zero at the λ-point[12].
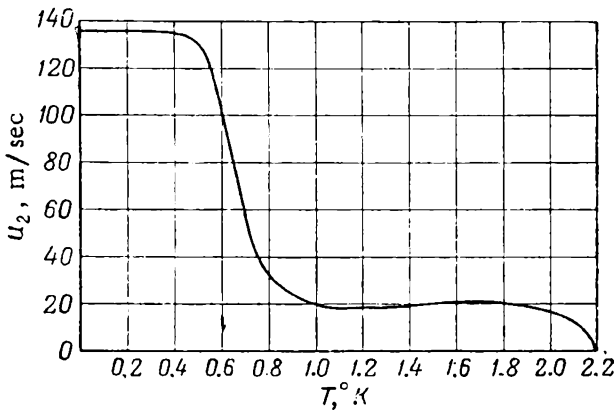
Fig. 5. Velocity of second sound in helium II.

Close to absolute zero, where $(\rho_n)_{\text{ph}} \gg (\rho_n)_r$ the specific heat and entropy can be expressed in terms of the "phonon" expressions (8.10) and (8.11) while the ratio ρ_n/ρ is given by (8.24). Substituting these expressions in (8.59), for u_2 we find

$$u_2 = \frac{u_1}{\sqrt{3}} \tag{8.60}$$

Thus, as the temperature approaches zero the velocity u_2 (as well

[12]An elementary derivation of the formula for the velocity u has been given by Gogate and Pathak [17].

as u_1) tends toward a fixed limit, while the ratio tends toward the value $1/\sqrt{3}$.

A curve of the temperature dependence of the velocity of second sound is shown in Figure 5.

In order to get a better understanding of the physical nature of of both forms of sound waves in helium II we consider a plane sound wave. In such a wave the velocities v_s and v_n and the variable parts T' and P' of the temperature and pressure are proportional to each other. We introduce the factor of proportionality by means of the formulas:

$$\mathbf{v}_n = a\mathbf{v}_s, \quad P' = bv_s, \quad T' = cv_s. \tag{8.61}$$

A simple calculation using equations (8.52) - (8.57) (Lifshits [18]), gives for second sound

$$a_1 = 1 + \frac{\alpha\rho}{S\rho_s} \frac{u_1^2 u_2^2}{(u_1^2 - u_2^2)}, \quad b_1 = \rho u_1, \quad c_1 = \frac{\alpha T}{C} \frac{u_1^3}{(u_1^2 - u_2^2)}, \tag{8.62}$$

and for first sound

$$a_2 = -\frac{\rho_s}{\rho_n} + \frac{\alpha\rho}{S\rho_n} \frac{u_1^2 u_2^2}{(u_1^2 - u_2^2)}, \quad b_2 = \frac{\alpha\rho u_1^2 u_2^3}{S(u_1^2 - u_2^2)}, \quad c_2 = -\frac{u_2}{S} \tag{8.63}$$

($\alpha = \frac{1}{\rho}\frac{\partial\rho}{\partial T}$ is the coefficient of thermal expansion). Quantities containing α are small compared with corresponding quantities which do not contain α .

We see that in a sound wave of the first type $\mathbf{v}_n \cong \mathbf{v}_s$, that is, in such a wave in the first approximation the liquid in each element of volume oscillates as a whole; the normal and superfluid parts move together. Obviously, this wave corresponds to the ordinary sound wave in ordinary liquids.

In a wave of the second type, however, $\mathbf{v}_n \cong -\frac{\rho s}{\rho n}\mathbf{v}_s$, that is the total density of material flow is

$$\mathbf{j} = \rho_s\mathbf{v}_s + \rho_n\mathbf{v}_n \cong 0.$$

Thus in a wave of second sound the superfluid and normal parts of the liquid oscillate "through" each other so that in the first

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approximation the "center of mass" in each elementary volume remains fixed, and there is no net flow. It is clear that this type of wave is peculiar to helium II.

Another important difference for these two types of waves is the fact that in a wave of second sound the amplitude of pressure oscillation is relatively small while the amplitude of the temperature oscillation is large [as is apparent from equation (8.63)]; in an ordinary sound wave the reverse is true.

RADIATION OF SOUND IN HELIUM II. The various methods of exciting sound waves in helium II have been considered by Lifshits [18]. It is found that the usual mechanical methods for the excitation of sound (oscillating solid bodies) are extremely ill suited for exciting second sound because the intensity of the second sound is extremely small as compared with the intensity of the ordinary sound, which is radiated simultaneously.

For example, we consider the radiation of sound waves by a plane which executes oscillations in a direction perpendicular to itself (we take this as the direction of the x-axis). We take the velocity v_s (directed along the x-axis) in the "first" and "second" radiated waves respectively in the form

$$v_s^{(1)} = A_1 \cos \omega \left(t - \frac{x}{u_1} \right), \quad v_s^{(2)} = A_2 \cos \omega \left(t - \frac{x}{u_2} \right)$$

(ω is the frequency of oscillation of the plane). The boundary conditions at the surface of the solid body require that the velocity of the body be equal to the velocity components perpendicular to its surface v_s and v_n (cf. the condition given in (8.39) at the fixed body). In the present case this condition yields the equations

$$A_1 + A_2 = v_0, \quad A_1 a_1 + A_2 a_2 = v_0$$

($v_0 \cos \omega t$ is the velocity associated with the oscillation of the solid plane), whence

$$\frac{A_2}{A_1} = -\frac{1 - a_1}{1 - a_2}.$$

As for any small oscillations, the time average of the potential energy is equal to the average kinetic energy. Hence the total energy density in the sound wave in helium II is

$$\rho_s \overline{v_s^2} + \rho_n \overline{v_n^2} = \frac{1}{2} A^2 (\rho_s + \rho_n a^2);$$

the energy flux (intensity) is obtained by multiplication by the appropriate sound velocity u. Using the values of a_1 and a_2 from equations (8.62) and (8.63), we find the following ratio for the intensities of the radiated waves of second and first sound:

$$\frac{I_2}{I_1} \simeq \frac{a^2 T}{C} \cdot \frac{u_2^3}{u_1} \tag{8.64}$$

(here it is assumed that $u_2 \ll u_1$ a condition which is valid down to very low temperatures). For example, at 2° K this ratio is $2 \cdot 10^{-6}$, that is, second sound is essentially not radiated.

However, in helium II there is another method of exciting sound waves which is peculiar to this liquid and which is impossible in ordinary liquids. This is the radiation of sound by a plane slab which executes oscillation in its own plane and in so doing "entrains" the normal part of the liquid[13]; radiation is also possible from a surface in which the temperature varies periodically as a function of time. This second method is especially suitable for obtaining second sound.

Suppose that the temperature of a plane solid surface varies in accordance with the relation $T' = T'_0 \cos \omega_0 t$. The conditions that the temperature be continuous and that the component of the total material flux perpendicular to the plane j be zero [cf. equations (8.39) and (8.40)] yield

$$\rho_s(A_1 + A_2) + \rho_n(a_1 A_1 + a_2 A_2) = 0, \quad c_1 A_1 + c_2 A_2 = T'_0.$$

The ratio $|A_2/A_1|$ is

$$\left|\frac{A_2}{A_1}\right| = \frac{\rho_n a_1 + \rho_s}{\rho_n a_2 + \rho_s} \simeq \frac{S}{a u_2^2}.$$

whence the ratios of intensities is found to be

$$\frac{I_2}{I_1} = \frac{C}{T a^2 u_1 u_2} \ . \tag{8.65}$$

At 2° K this quantity is $5 \cdot 10^3$ and it becomes larger at lower temperatures. Thus, as far as intensity ratios are concerned, in this

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[13]This method was proposed by Kapitza.

case only second sound is radiated. The ratios of the pressure amplitudes and temperature amplitudes are

$$\frac{P'_2}{P'_1} = \frac{o_2 A_2}{b_1 A_1}, \qquad \frac{T'_2}{T'_1} = \frac{c_2 A_2}{c_1 A_1},$$

whence

$$\frac{P'_2}{P'_1} = \frac{u_2}{u_1}, \quad \frac{T'_2}{T'_1} = \frac{C}{a^2 T u_1 u_2}. \tag{8.66}$$

At 2° K it turns out that $P'_2/P'_1 = 0.1$ so that the conditions for the observation of second sound by pressure variations are still not favorable. However, the conditions for the observation of second sound by variations in temperature turn out to be very favorable.

EFFECT OF IMPURITIES ON THE PROPAGATION OF SECOND SOUND IN HELIUM II. When there are impurity atoms present in helium II (including the isotope He^3), at sufficiently low temperatures the effect of the impurities on the thermodynamic properties of the liquid become predominant (cf. Section 2) — a situation which obviously must also have an effect on the propagation of second sound in helium II (this problem has been considered by Pomeranchuk [19] in a work which will not be considered in detail here).

We may note here that a calculation of the velocity of sound in the presence of impurities requires not only a determination of the effect of the impurities on the thermodynamic properties of the helium but also the introduction of several modifications in the equations of motion of the liquid. First of all, it is necessary to take account of the "osmotic pressure" of the impurities; in this connection the expression in the curly brackets in equation (8.38) must be supplemented by the term kTN_{imp} where N_{imp} is the number of impurity atoms per gram of helium. Furthermore, the system of hydrodynamic equations must be supplemented by a "conservation equation" for the number of dissolved particles:

$$\frac{\partial(\rho N_{imp})}{\partial t} + \mathrm{div}\,(\rho N_{imp} v_n) = 0$$

(here it is assumed that the impurity moves with the normal mass).

At this point we shall not dwell on the general extremely complicated expressions which are obtained for the velocity of second sound in the presence of impurities [19]. A basic feature of the result is the fact that at sufficiently low temperatures (the lower the temperature, the smaller the impurity concentration) the velocity of second sound, having reached a maximum, falls off, approaching the constant limit $c/\sqrt{3}$, as in pure helium II. In the temperature region in which the thermodynamic properties are determined by the impurities, the velocity of second sound obeys the relation

$$u_2 = \sqrt{\frac{5\,kT}{3\,\mu'}}$$

(if $p'_0 = 0$) or

$$u_2 = \frac{3\,kT}{p'_0}$$

[if $p'_0 \neq 0$; p'_0 and μ' are the parameters in equation (8.29)]. Both of these formulas can be shown to have a simple physical significance: they correspond to the propagation of second sound "by the impurity" alone, i.e., just as in a single-atom gas, the particle energy of which is determined by equation (8.29).

The reduction in u_2 continues up to a point which, although close to absolute zero, does not exhibit quantum degeneracy of the "impurity gas" or interaction of the impurity particles with each other.

SCATTERING OF LIGHT IN HELIUM II. The scattering of light in helium II should exhibit certain features by which it differs from the scattering of light in ordinary liquids (Ginzburg [20]). Although these effects are still beyond the capabilities of experimental observation they are of great theoretical interest.

First of all, it should be pointed out that the expectation of an anomalously strong scattering of light in helium II, which has been proposed in the literature [21], [22], [23], is completely without basis. These proposals are based on the scattering of light by an ideal Bose-Einstein gas which should exhibit an extremely pronounced scattering of light close to the condensation point. However, since the Bose-Einstein condensation has little meaning with regard to the properties of helium II there is no basis for attributing the optical properties of such a gas to helium II.

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The total intensity of scattered light in helium II is determined by the well-known general formula for scattering due to density fluctuations in an isotropic body (liquid or gas);

$$I = \frac{\pi^2 V}{2\lambda^4} \left(\rho \frac{\partial \varepsilon}{\partial \rho} \right)^2 \frac{1}{\rho} \left(\frac{\partial \rho}{\partial P} \right)_T kT (1 + \cos^2 \varphi).$$

Here I is the ratio of the intensity of the scattered light (computed for unit solid angle) to the flux density of the incident light, V is the scattering volume, λ is the wave length of the light in vacuum, $\varepsilon = n^2$ is the square of the index of refraction and φ is the scattering angle (the light is assumed to be unpolarized).

An estimate made on the basis of this formula indicates that in helium close to the λ-point $I \sim 2 \cdot 10^{-8}$ (for $V = 1$ cm^3, $\lambda = 4 \cdot 10^{-5}$ cm), which is approximately equal to the intensity of light scattering in air at room temperature[14].

As is well known, density fluctuations can be divided into adiabatic and isobaric fluctuations[15]. Fluctuations of the first type are propagated with the velocity of sound in the liquid. Scattering on these fluctuations leads to a spreading of the scattering line into the so-called Mandel'shtam-Brillouin doublet with a relative distance between the components $\frac{\Delta\omega}{\omega} \sim \frac{u}{c}$ (c is the velocity of light and u is the velocity of sound). However, in an ordinary liquid the isobaric fluctuations (fluctuations in entropy) are not propagated and are dissipated by virtue of thermal conductivity. Scattering on these fluctuations does not affect the frequency of the light. As a result the scattering line is a triplet with an undisplaced component at the center.

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[14]The absence of an anomalously strong scattering was actually established experimentally by McLennan.

[15]Writing a small change in density in the form

$$\Delta\rho = (\partial\rho/\partial P)_S \Delta P + (\partial\rho/\partial S)_P \Delta S$$

and understanding that the fluctuations in pressure and entropy are independent $\Delta P \Delta S = 0$ we find that the mean square fluctuation $\langle \Delta\rho^2 \rangle$ can be given in the form of a sum of the adiabatic pressure fluctuations and the isobaric entropy fluctuations (with appropriate coefficients).

In helium ll the situation changes because the entropy fluctuations are also propagated in the liquid — with the velocity of second sound. Hence, in addition to the ordinary doublet (with a relative splitting $\frac{\Delta\omega_1}{\omega} \sim \frac{u_1}{c}$) there should also be observed a doublet whose position lies within the first one (with a relative splitting $\frac{\Delta\omega_2}{\omega} \sim \frac{u_2}{c}$).

Thus, as a result, the scattering line should be a quadruplet. It should be kept in mind, however, that not only is the interval between the components of the "anomalous" doublet very small but its intensity (as can be shown by an appropriate calculation) is negligibly small as compared with the intensity of the normal doublet.

VISCOSITY OF HELIUM II. A special problem in the theory of helium II is that of computing its viscosity (the viscosity of the "normal part"). This problem has been considered by Landau and Khalatnikov [24]. Here, we shall present only a brief summary of their results, omitting the complicated and lengthy calculations.

In treating viscosity we encounter effects of kinetic character associated with the processes which arise when equilibrium is established in the phonon and roton "gas". A calculation of viscosity requires, first of all, an investigation of the various types of elementary collision processes between the particles of this gas and a computation of the effective cross sections for these processes[16].

The transfer of momentum in helium II (when there is a velocity gradient) is realized both by phonons and rotons. Consequently, the viscosity may provisionally be represented as a sum of two parts — a phonon part and a roton part. First we shall consider the roton part of the viscosity.

In accordance with the well-known formula from the kinetic theory of gases, the order of magnitude of the viscosity of a gas is given by the expression

$$\eta \sim mvln$$

where m is the mass of a molecule of the gas, v is its mean ther-

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[16]We may make mention of the fact that Akhiezer and Pomeranchuk [25] have considered collisions of slow neutrons with rotons and phonons for the purpose of determining the intensity of neutron scattering in helium II (it was found that helium II is essentially "transparent" to slow neutrons).

mal velocity, l is the length of a molecular free path and n is the number of molecules per unit volume. It is apparent from equation (8.26) that the role of the particle mass in the roton gas is played by the quantity $p_0^2/3kT$ (the roton part of the density ρn is obtained by multiplying this quantity into the number of rotons). The mean thermal velocity of a roton is given by $v^2 \sim \overline{(p - p_0)^2}/\mu^2 \sim kT/\mu$. Thus, for the roton viscosity we have

$$\eta_{\mathbf{r}} \sim \frac{p_0^2}{\sqrt{\mu kT}} l n_{\mathbf{r}}. \tag{8.67}$$

To determine the length of the roton free path we must consider the various kinds of collisions which the rotons experience: 1) elastic collisions between rotons, 2) elastic collisions between rotons and phonons, 3) inelastic collisions between rotons, accompanied by the emission (or absorption) of new rotons or phonons. Calculations indicate that the probability for various inelastic scattering collisions between rotons is considerably smaller than the probability for elastic collisions, as might be expected. As far as collisions of rotons with phonons, however, at temperatures which are not too low the probability for these processes is considerably smaller than the probability for elastic scattering of rotons on rotons because of the fact that the number of phonons is very small compared with the number of rotons. It is only at sufficiently low temperatures (starting at approximately 0.5° K) that roton-phonon collisions start to dominate because of the marked reduction in the number of rotons as compared with the number of phonons. It should be kept in mind, however, that at temperatures below 0.5° K the usual conception of viscosity tends to lose its significance since the free path length for phonons and rotons becomes (as can be shown by calculation) of the order of 1 cm, that is to say, comparable with the dimensions of the apparatus with which viscosity measurements are usually carried out.

Thus, remaining in the region of very low temperatures we can show that the mean path length for the rotons is determined basically by elastic collisions between rotons. Denoting the effective cross section for these collisions by $\sigma_{\mathbf{rr}}$, we have $l \sim 1/n_{\mathbf{r}}\sigma_{\mathbf{rr}}$. The effective cross section $\sigma_{\mathbf{rr}}$ for rotons with average thermal velocities turns out to be inversely proportional to $\sqrt{T}$; substituting

in equation (8.67) we find that the roton part of the viscosity is a constant, independent of temperature[17]

$$\eta_r = const \qquad (8.68)$$

The calculation of the phonon part of the viscosity is especially complicated. A careful analysis of the various elastic and inelastic collision processes between phonons and between phonons and rotons leads to the following results.

At temperatures above approximately $0.8°$ K the main role in the transfer of momentum is played by elastic collisions between phonons and rotons. The effective cross section σ_{rr} for these collisions (for phonons with mean thermal energies[18]) can be computed and turns out to be proportional to T^4. Qualitatively, this dependence can be obtained if it is noted that because of the large value of the roton momentum as compared with the mean phonon momentum

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[17]In order to make an exact calculation of the probability for roton-roton collisions it would be necessary to know the laws which govern their interaction. The existing theory does not afford the possibility of reaching any definite conclusions in this regard. However, in order to determine the temperature dependence of the effective cross section it is possible to use an approach used in the theory of neutron scattering, writing the interaction energy U for two rotons in the form of a δ-function in the difference in their coordinates: $U = U_0\delta(\mathbf{r_2}-\mathbf{r_1})$, where U_0 is some constant.

The probability for collision of two rotons (in a given volume Ω), computed by means of the theory of excitations, is a constant which depends only on p_0, μ, U_0 (and $\hbar$). This result, however, might be expected beforehand on the basis of the fact that the roton momentum is close in absolute value to the constant p_0 (this is also taken into account in the calculation). The effective cross section is obtained by multiplying the probability by Ω/v. whence the relation given in the text is obtained

An exact expression for the viscosity (in terms of the constants p_0, μ, U_0) can be obtained by solving the appropriate kinetic equation.

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[18]It is important that the energy exchange between phonons and the establishment of energy equilibrium take place rapidly, as has been shown by investigation.

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($p_0 \gg kT/c$) scattering of phonons on rotons should be analogous to the scattering of sound waves on a fixed solid sphere. As is well known, the effective cross section for this type of scattering is proportional to the fourth power of the sound frequency. Hence we may conclude that the effective cross section for the scattering of phonons on rotons is proportional to the fourth power of the wave vector (i.e., the momentum) of the phonon, whence (in view of the relation $p \sim kT/c$) we obtain the temperature dependence given above.

In the gas-kinetic formula $\eta \sim mv/n$ it is now necessary to replace the product mn by the phonon part of the density ρ_n [equation (8.24)] while the velocity v is the velocity of sound c. Writing the mean path length $l \sim 1/\sigma_{ph\,r}\, n_r$, we have

$$\eta_{ph} \sim \frac{k^4 T^4}{\hbar^3 c^4 \sigma_{ph^n r}} \,.$$

In accordance with equation (8.14) the number of rotons depends on the temperature in accordance with the relation $\sqrt{Te}^{-\Delta/kT}$ Taking account of the fact that $\sigma_{ph\,r} \sim T^4$, we find the following relation for the temperature of the phonon viscosity (at $\overline{T}$ 0.8° K):

$$\eta_{ph} \sim T^{-1/2} e^{\Delta/kT}. \tag{8.69}$$

At temperatures below approximately 0.8° K, because of the marked reduction in the number of rotons the basic role is played by phonon-phonon scattering. In this temperature region calculations indicate the following temperature dependence for the viscosity:

$$\eta_{ph} \sim T^{-5}. \tag{8.70}$$

Thus we see that the phonon part of the viscosity increases rapidly as the temperature is reduced — first, exponentially and then, as $1/T^5$. It is important to note that in both indicated temperature regions it is possible not only to determine the temperature dependence of the phonon part of the viscosity but also to make a complete calculation of its absolute value with no undetermined con-

stants in the expression which is obtained (such as the constant U_0, which characterizes the interaction of two rotons, cf. the footnote at bottom of p. 48). This situation makes it possible to carry out a comparison of the theoretical and experimental results without introducing new parameters. However, it should be kept in mind that in the expression for viscosity there appear not only quantities such as μ, Δ, p_0 and c but also their derivatives with respect to the density of the liquid. These derivatives can be determined from the available experimental data using the variation in the velocities of first and second sound with pressure; however, at the present time the accuracy with which these measurements can be carried out is relatively poor.

Systematic measurements of the viscosity of helium II have been carried out recently by Andronikashvili. He has observed that in the region from 1.9 to 1.6° K the viscosity remains more or less constant while a further reduction in temperature results in a rapid increase in viscosity (the measurements have been carried down to 1.3° K[19]). If, from the experimental values of the viscosity we subtract the value of η_{ph} calculated from the theoretical formula, a quantity which is approximately constant is obtained; this may be identified with the roton part of the viscosity[20]. Thus, completely satisfactory agreement obtains between theory and experiment.

[19]In the region from the λ-point to 1.9° K the viscosity falls off as the temperature is reduced. In this region, however, the notion of a roton gas and a phonon gas loses its significance and a theoretical calculation of the viscosity becomes impossible.

[20]Comparing the value of the viscosity obtained in this manner with the theoretical expressions, we can determine the constant U_0 which characterizes the interaction between rotons. On the other hand, using this same constant it is possible to express the coefficient in the "van-der-Waals correction" which may be introduced for the purpose of taking account of the departures from an ideal roton gas which arise as the λ-point is approached (ρ_n is written in the form $\rho_n = (\rho_n)_r [1 + \alpha(\rho_n)_r]$ where $(\rho_n)_r$ is the "ideal gas" expression (8.25) while α is a constant). This makes possible an independent determination of U_0, starting from the data on the ratio ρ_n/ρ for temperatures in the region of 1.9 − 2.0° K. The values of U_0 determined by both methods are found to be of the same order of magnitude.

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THE TISZA THEORY OF HELIUM II. As has been mentioned in Section 1, in the first papers by Tisza, following London, helium II was considered a degenerate ideal Bose-Einstein gas. In his more recent papers [9], [10] the discussion is concerned with a "quantum Bose-Einstein liquid" but the difference lies only in certain statements as to the wave function of a system of many particles; then this author again describes atoms which are "condensed" to the lowest state and excited atoms (the excitation energy of which are called "translational quanta" by the author). This analysis of the motion of individual atoms in a system of strongly interacting particles (liquid) is at complete variance with the basic principles of quantum mechanics. Indeed, a completely unavoidable consequence of quantum mechanics is the need for introducing, for any weakly excited macroscopic system, the notion of elementary excitations (this lies at the basis of the microscopic theory proposed by Landau) which describe the "collective" motion of the particles; each such excitation can be assigned a definite energy ε and effective momentum p (neglecting for the time being the functional relation $\varepsilon(p)$, that is, the form of the spectrum).

Although Tisza's work contains no microscopic analysis it is still of indisputable value; its importance lies in the introduction, independently of Landau[21], of the notion of a macroscopic description of helium II through separation of its density into two parts and the introduction of two velocity fields. On the basis of these considerations, even as early as 1938 Tisza was able to predict the existence of a second kind of sound wave in helium II (called a temperature wave by this author).

However, Tisza could not formulate a quantitatively correct and self-consistent hydrodynamic and thermodynamic theory for helium II. A more recent paper [10] is also devoted mainly to a macroscopic theory. It should be noted that a considerable part of this paper is devoted to a thermodynamic derivation of the hydrodynamic equations, the boundary conditions for these equations, a thermodynamic formula for the velocity of second sound, a discussion of experi-

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[21]The detailed paper by Tisza (1940) [9] was first obtained in the USSR in 1943 because of wartime conditions while the short note in Compt. rend., Paris [8] remained unnoticed at that time.

ments on the measurement of viscosity etc., without mentioning, however, that all this had been done by Landau (in the earlier Tisza papers these formulas were not given).

The basic error committed by Tisza lies in a misunderstanding of the role of the phonons. Tisza excludes phonons from the normal part of the density of helium II, arguing that the phonons are "associated with the liquid as a whole" and with "an excited atom of helium in translational states of the Bloch type" (comprising the normal part of the liquid in this theory). This statement and the argument are fundamentally incorrect. The phonons and the rotons are associated in equal degree with the "collective" motion of the atoms of the liquid (although the phonons are characterized by longer wave lengths than the rotons) and, in accordance with quantum mechanics, can be considered as "quasi particles", having momentum. Without dwelling on the fact that the entrainment of phonons (as well as rotons) by moving walls has been demonstrated by rigorous thermodynamic considerations, presented in Section 2[22], it is obvious *a priori* that, for example, in the flow of helium through a narrow slit the phonons will unavoidably be scattered on the walls, (not mentioning phonon-phon collisions, the probability of which can be computed hydrodynamically and which is by no means zero).

A further error is found in the statement by Tisza that there is a proportionality between the entropy and the normal density of helium. This relation cannot be obtained by thermodynamic means

[22]The objections of Tisza to these considerations [that they are "used to obtain information on a kinetic coefficient (viscosity) starting from equilibrium considerations"] are the result of a misunderstanding. It is generally known that uniform rotation is amenable to strict thermodynamic analysis; in Section 2 this analysis is applied only to demonstrate the entrainment of the liquid as a whole by the rotating container (no conclusions as to the magnitude of the viscosity can be drawn and no attempt was made to do so).

Another misunderstanding is found in the remark that rotons should be "associated with a definite mass, enclosed in a volume in which the roton velocity is different from zero"; the incorrectness of this statement is clear from the foregoing:

and, from the formulas given in Sections 1 and 2, it is obvious that it does not, in fact, exist[23].

In deriving the hydrodynamic equations Tisza has committed a number of errors, as a result of which, in their exact form his equations do not even obey the conservation laws. Only the first-approximation equation (in these equations terms of second order in velocity are neglected) are correct and these coincide with the Landau equations. Naturally, the formula for the velocity of second sound is identical with the Landau formula (8.59). However, because of the misinterpretation of the phonon role, mentioned above, Tisza excludes the phonon part from the entropy S which appears in this formula. Tisza obtains the following relation for the temperature dependence of the remaining part of the entropy:

$$S = S_0 \, (T/T_0)^r$$

(T_0 is the temperature of the λ-point, the exponent r is taken as 5.5 to obtain agreement with the empirical values of the entropy[24]). Using the proportionality proposed by Tisza between the entropy and normal density, he obtained the following expression for the velocity of second sound:

$$u_2 = 26 \, \sqrt{\frac{T}{T_0} \left[1 - \left(\frac{T}{T_0} \right)^{5.5} \right]} \quad \text{m/sec.}$$

When $T \to 0$ this formula yields $u_2 \to 0$, that is to say, the velocity of second sound approaches zero rather than the finite value $c/\sqrt{3}$ which follows from the Landau theory[25].

[23]For incidental reasons the temperature dependence of the roton parts of ρ_n and the entropy are found to be similar [cf. equations (8.16) and (8.26)]. This is the reason that Tisza was able to obtain a good fit between the formulas and the experimental values of the entropy and velocity of second sound in the temperature region above approximately 1.3° K.

[24]A theoretical calculation of the temperature dependence of the thermodynamic quantities in helium ll could not be carried out by Tisza because of the absence of a microscopic theory in his work.

[25]In a recent paper [J. Exptl. Theoret. Phys. (USSR) 18, 857 (1948)] Peshkov has observed experimentally a minimum in the velocity of second sound at a temperature of 1.1°K after which u_2 starts to increase – in complete agreement with the Landau theory and in contrast to the statement made by Tisza.

The Tisza paper also contains certain observations as to the behavior of the viscosity of helium II. However, his consideration of this problem, which is actually the most complicated of all, (cf. above), is extremely naive. They contain remarks as to the need for distinguishing between viscosity of a "liquid type" and viscosity a "gaseous type"; as a result, Tisza reached the conclusion that viscosity is reduced as the temperature is reduced, a result which is in complete disagreement with the measurements of Andronikashvili mentioned above.

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CHAPTER IX

SUPERFLUIDITY (EXPERIMENTAL DATA)

E. L. Andronikashvili

Section 1. Motion of Helium II Due to the Influx of Heat

INTRODUCTION. Of all the properties of helium II described in Chapter VI, the most remarkable are the extraordinarily high transport of heat through narrow heat conductors filled with this liquid and the very low viscosity observed in the flow of this liquid through narrow capillaries and slits. The first effect was found by Keesom and Keesom [1] while the second was discovered in the experiments carried out by Kapitza [2] and later in the experiments of Allen and Misener [3]. Because of the unusual behavior of helium II Keesom called it a "super heat conductor." On the other hand, because of the absence of any measurable viscosity, Kapitza called helium II at temperatures below the λ-point a "superfluid."

It was apparent that both of these properties were intimately related and that in all probability one of them was a consequence of the other. This point of view, first proposed by Kapitza, was at the basis of his further research into the properties of helium II [4] which helped to unravel the nature of this relation and indicated what is probably the best approach to the experimental investigation of the properties of quantum liquids.

HEAT TRANSPORT IN HELIUM II MOVING IN A CAPILLARY. The dependence of heat transport on heat load [5] tends to support the Kapitza hypothesis that the heat conductivity of helium II is not a true heat conductivity but is due to its high fluidity and the

consequent possibility of the production of extraordinary convection currents. Anything which affects these currents is found to have an effect on the heat conductivity.

This effect is most easily demonstrated when helium II which fills a capillary is, by some means or other, forced to move with respect to the walls. Such experiments were carried out successfully by Kapitza [4].

Even slight variations in the pressure in a device such as that shown in Fig. 6, connected to the helium system, are found capable of reducing the heat transport tremendously.
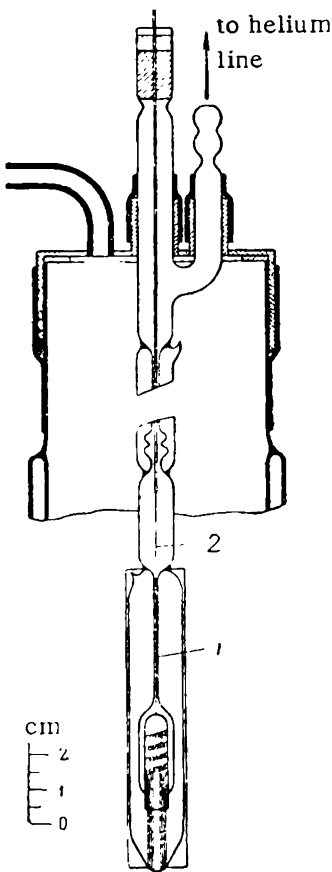

Fig. 6. System for studying the effect of pressure variation and agitation on heat transport.
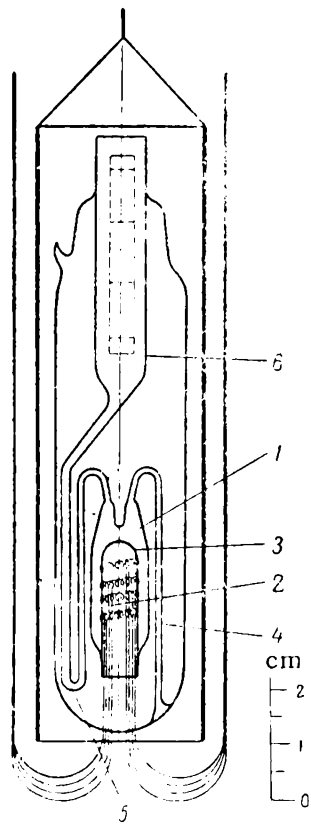
Fig. 7. System for studying the effect of helium flow on heat transport.

The flow of helium through a capillary, in the presence of an electric heater also increases the temperature gradient significantly, i.e., reduces the heat transport.

The apparatus with which Kapitza investigated this effect is shown in Figure 7. To a vessel 1 containing a heater 2 and a bronze thermometer 3 are connected symmetrically two capillaries 4 and 5. These capillaries are 5 cm in length and have an internal diameter of 0.36 mm. The vessel is connected to the helium bath through the capillaries; the connection is either direct (capillary 4) or through a thick glass tube and container 6, on the walls of which fiduciary marks are scratched
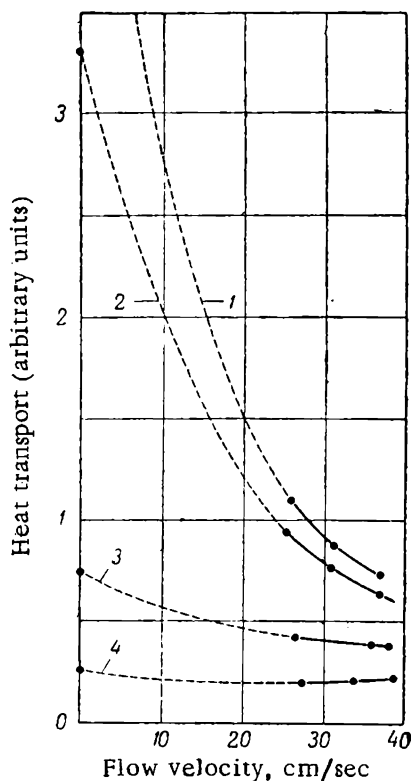

Fig. 8. Heat transport as a function of flow velocity. $T = 1.58°$ K. The curves refer to the following loads: 1) 0.175, 2) 0.39, 3) 0.70, 4) 1.09 watts/cm^2.
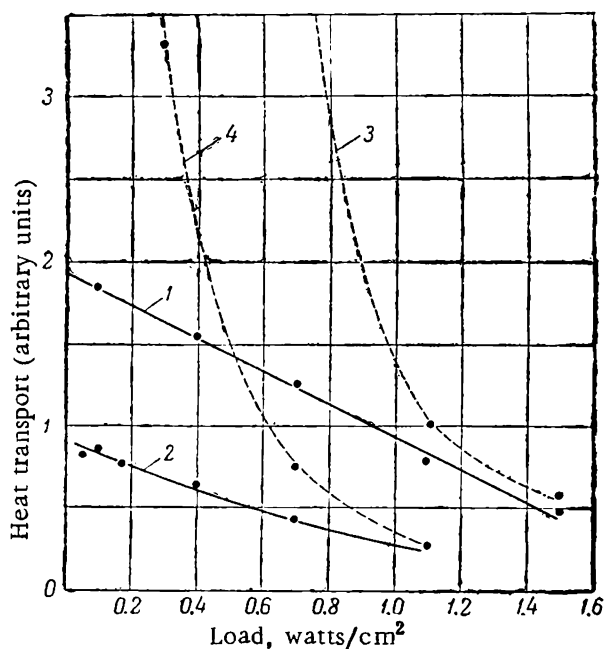
Fig. 9. Heat transport as a function of load in helium at rest (curves 3 and 4 and in motion). Curves 1 and 3 are for $T = 1.85°$ K, curves 2 and 4 are for $T = 1.58°$ K.

at equal distances. The entire system is surrounded by a vacuum housing and a copper radiation shield. While pouring helium into container 6 and observing the rate of flow from top to bottom or submerging the empty bulb in the bath and observing the flow of helium into container 6, Kapitza simultaneously measured the power in the heater and the difference in temperature at the two ends of the capillary. The rate of flow was measured using a timer and the marks on container 6. The dependence of heat transport on flow rate is shown in Figure 8.

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The curve shows that at high loads the heat transport is essentially independent of the rate at which helium flows through the capillary whereas at low loads the dependence becomes very marked. These characteristics are evident in Figure 9 which shows the dependence of heat transport on load in helium at rest (dashed curves) and helium flowing at a velocity of 35 − 40 cm/sec (solid curves).

It may be noted that in these experiments it was also observed that at high loads a gas bubble forms in the helium II; the formation of this bubble is accompanied by a sharp change in the temperature difference at the ends of the capillary. This effect indicates the possibility of strong superheating of the liquid.

It is also found possible to affect the heat transport in helium II in a capillary by setting the helium into rotational motion. A device similar to that shown in Figure 6 is used for this purpose. The heat transport is measured in capillary 1 of length 4 cm and 0.62 mm in diameter in which a wire 2, 0.5 mm in diameter, is rotated. The wire is rotated at 1900 rpm. The rotational motion of the wire induces rotational motion in the helium in the capillary.
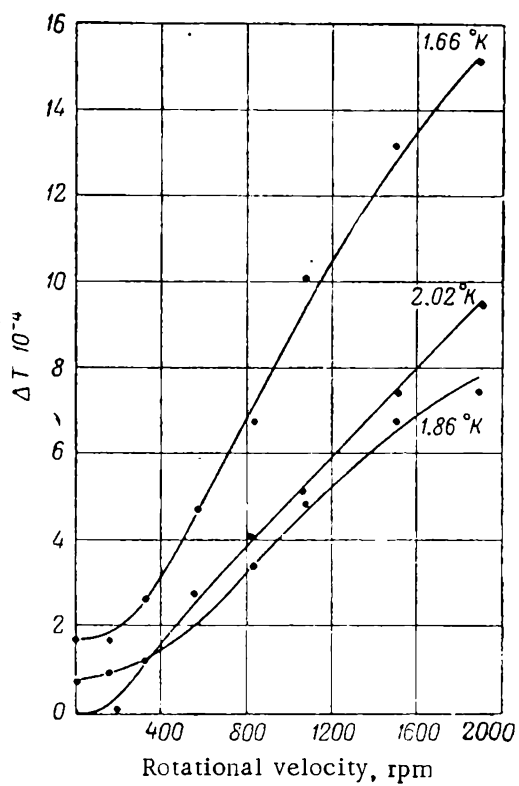
Figure 10 shows the temperature difference arising at the ends of the capillary for a fixed load as a function of the rotational velocity.

Fig. 10. Effect of rotation of helium II in a capillary on heat transport.

It has been established in control experiments that rotation of the wire with no flow through the heater produces no increase of the temperature inside the vessel. Thus, the temperature gradient at the ends of the capillary is due entirely to the reduction in heat transport.

NATURE OF THE JET AND THE JET PROFILE. From the experiments described above it is obvious that the heat transport is to be associated with certain currents and that any disturbance of these currents due to external agencies such as flow, rotation, pressure variation, etc., reduces the transport of heat. These same conclusions are supported by experiments in which high heat loading itself leads to a reduction in heat transport.

On the other hand, the extraordinarily high heat transport cannot be explained by any ordinary convection mechanism since (and this follows from elementary calculations carried out by Kapitza) it would have to be assumed that the velocity associated with the convection motion of the liquid is 1000 meters/sec, a figure which is clearly untenable.

In order to establish the nature of the currents responsible for the transport of heat in helium II it is desirable to observe these currents directly; this was done by Kapitza.

The experiment in point is shown in Figure 11. The vessel 1 and the capillary 2, 6 mm long with an internal diameter of 0.5 mm which is attached to the vessel, comprise a miniature Dewar flask. The cross section of the capillary is uniform over its entire length. A constantan heater and a bronze thermometer are suspended inside the bulb. Opposite the open end of the capillary, on a light yoke 4 is suspended a vane 3. The balance is attached to a glass rod 5, which in turn is fastened to a quartz fiber. At the end of the rod is a mirror which serves to indicate the angle through which the fiber

Fig. 11. Device for studying the profile of a thermal excitation jet.
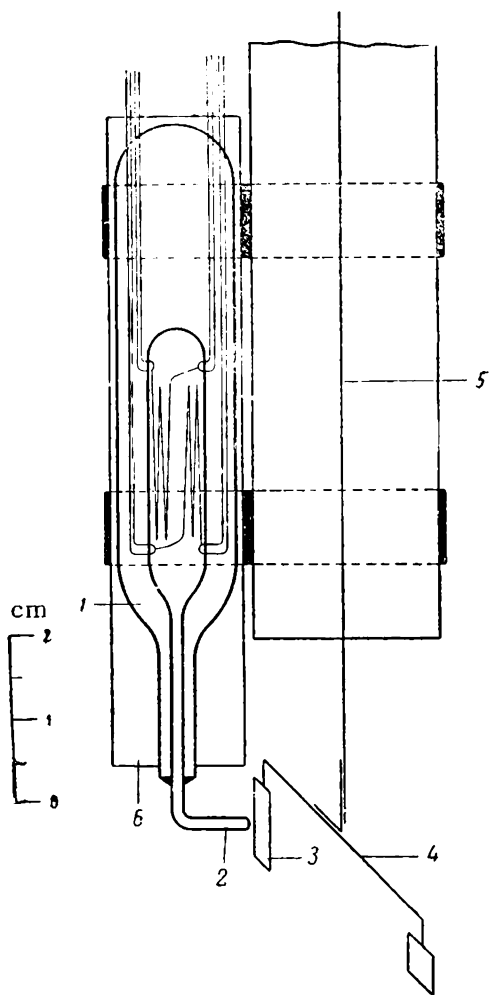
is twisted. The tortional constant, determined in preliminary experiments, is $8.96 \cdot 10^{-2}$ dynes/rad. The vane is made from thin silver foils and is balanced by a counterweight at the other end of the yoke. A copper shield 6 protects the helium inside the bulb against heating.

Submerging the system in helium II and locating the vane opposite the exit aperture of the capillary one observes a deflection of the "rotor" when the heater inside the vessel is on. The experiments show that under these
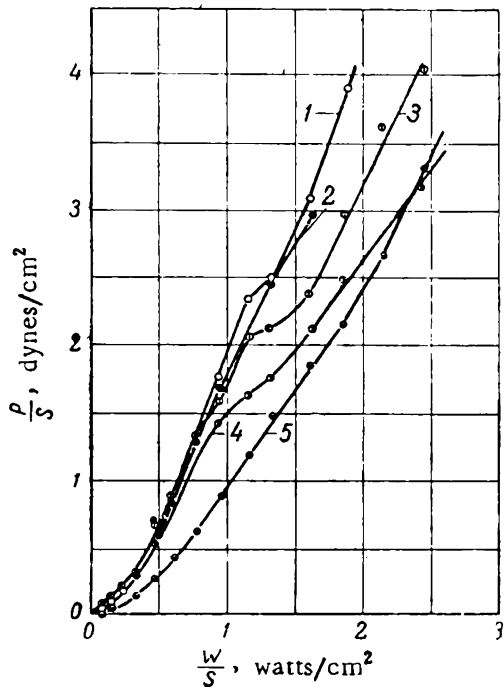
Fig. 12. Pressure of a thermal excitation jet obtained with the device shown in Fig. 11.

Fig. 13. Temperature differential at the ends of the capillary obtained with the device shown in Fig. 11. watts/cm²

conditions a pressure acts on the vane and that the pressure is only slightly dependent on the distance from the end of the capillary. (This distance varies between 0.5 and 15 mm.)

By displacing the vane in a direction transverse to the axis of the capillary it is easily shown that the stream which issues from the end of the capillary is highly directional. In Figure 12 is shown graphically the pressure of the jet on the vane as a function of the heater load, referred to unit area of the exit aperture of the capillary. Curves 1, 2, 3 and 4 were taken at a temperature of 1.7° K at distances of 0.5, 1.4, 7.0 and 14.4 mm, respectively. Curve 5 was

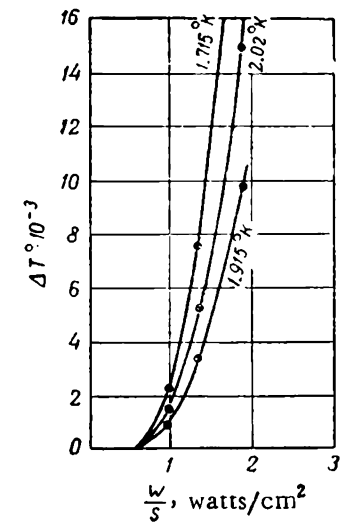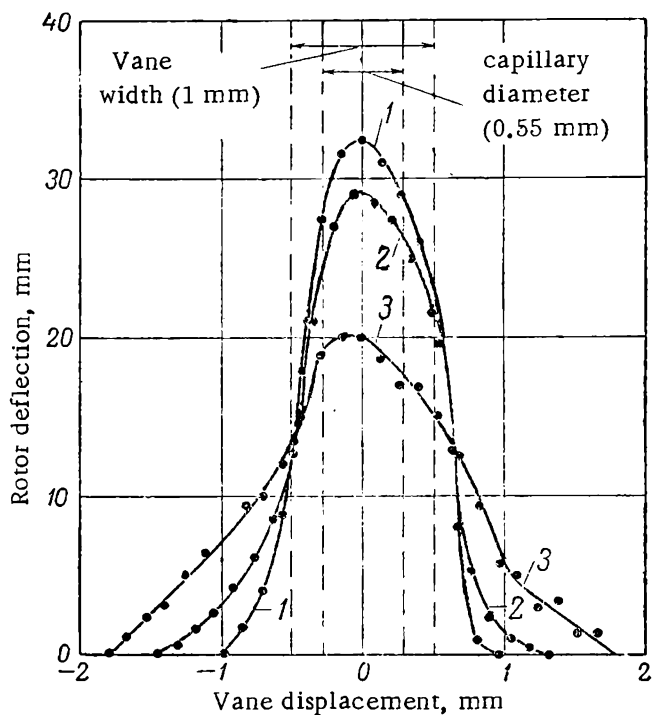Fig. 14. Profile of the thermal excitation jet at $T = 1.6°$ K and a load of 0.35 watts/cm². 1) at a distance of 1 mm, 2) 8.2 mm, 3) 15.4 mm.

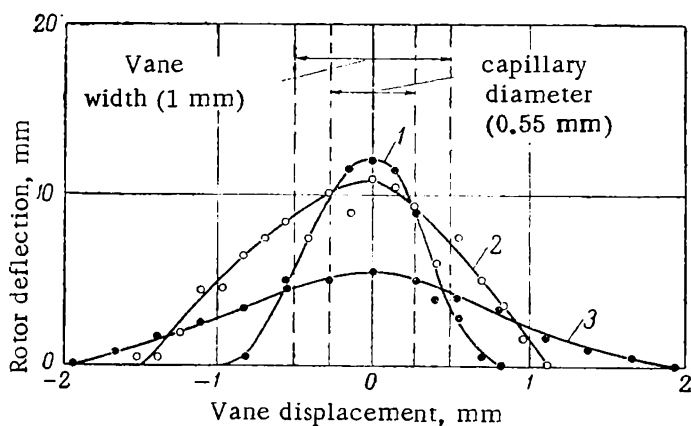Fig. 15. Profile of the thermal excitation jet at $T = 2.02°$ K and a load of 0.62 watts/cm². The distances are the same as in Fig. 14.

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taken with T = 1.915° K, with a distance of 0.5 mm between the end of the capillary and the vane.

It is noteworthy that in the region close to the origin the variation of pressure with heat load is quadratic. In this region the deflection of the rotor is completely stable. An increase in the heat load causes instabilities in the jet flow.

As is evident from the curves, the pressure of the jet on the vane increases as the temperature is reduced.

Figure 13 shows the increase in the temperature inside the vessel as a function of heat load. At low temperatures the temperature change lies beyond the limits of the sensitivity of the bronze thermometer although the pressure at the vane is easily measured in this region — a very remarkable situation which will be explained below.

The pressure distribution over the cross section of the jet is shown in Figures 14 and 15 for temperatures of 1.6 and 2.02° K, respectively.

Comparing the total width of the vane (1 mm), the end of the capillary (0.55 mm), and the width of the jet (Figures 14 and 15) one is easily convinced that within the limits of the observational errors the cross section of the jet is essentially the same as the cross section of the capillary. When the distance from the end of the capillary to the vane is increased, the transverse dimensions of the jet change slightly at low temperatures; it is only at distances of the order of 15.4 mm that deformation of the jet can be noticed. The deformation of the stream becomes more noticeable at higher temperatures.

REACTION OF THE JET. The experiments described in the preceding paragraphs indicate that a jet issues from the vessel when heat is applied inside. It is apparent that against this current there should move a countercurrent, possibly in some other energy state. The following experiments were performed by Kapitza in order to verify the existence of this countercurrent.

A small glass vessel 1 (Figure 16), in the form of a Dewar flask with a capillary 1 cm long and an aperture diameter of 0.66 mm, is suspended on yoke 2 and balanced by disk 3. The disk also serves as a damper. The yoke is attached to the suspension system

described above, consisting of a glass rod 6, a mirror and a quartz fiber with a tortional constant of $8.96 \cdot 10^{-2}$ dynes/rad.

The vessel contains a heater; the heater current flows through thin silver ribbons 4 and 5. The tortional constant for these strips is determined from the change in the oscillation period of the system when they are removed.

In the initial experiments the flow was not hindered and in flowing from the end of the capillary with heat applied in the vessel, the current could exert a reactive force on the suspension system. Figure 17 shows the dependence of the reactive force, referred to unit cross sectional area of the capillary, on the heat load. The following features are apparent: a nonlinear relation at small loads, an increase in pressure as the temperature is reduced, a region of unstable motion. However, the results of this experiment still do not show that the effect is due solely to the flow from the vessel.

Fig. 16. Device for studying the reactive force of the jet.

Hence, in later experiments a vane 7, 4 mm in diameter, was attached to the vessel, 1 mm behind the end (Figure 16). The fact that the reactive force due to the flow from the vessel was equal to the pressure on the vane is evidence that the countercurrent flows through the capillary without exerting any pressure on the vessel.

To a first approximation the experiment supports this assumption: the ordinates corresponding to the curve (lowest on Figure 17) are less than 5 - 7 per cent of the ordinates for a curve taken at the same temperature but without the vane. As indicated by the author, this deviation lies within the experimental errors and thus the entire reactive effect can be attributed to the flow from the vessel.

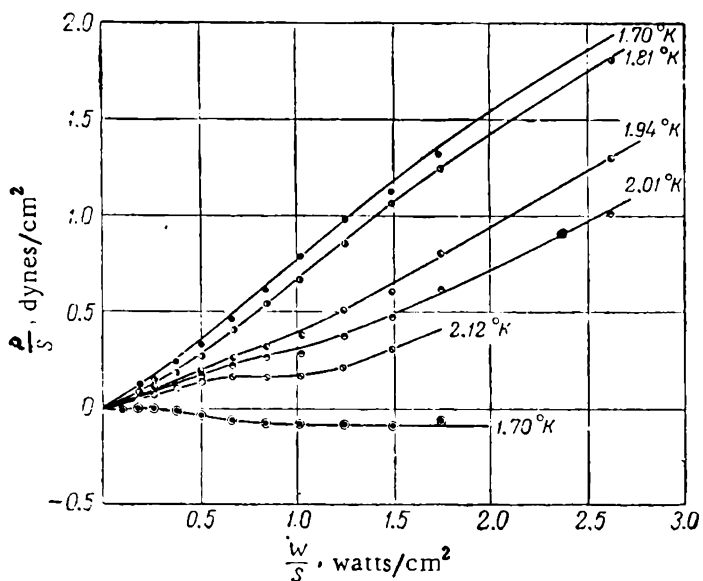Fig. 17. Reaction of the jet on the vessel. The lower curve applies when the capillary is covered by a disk.
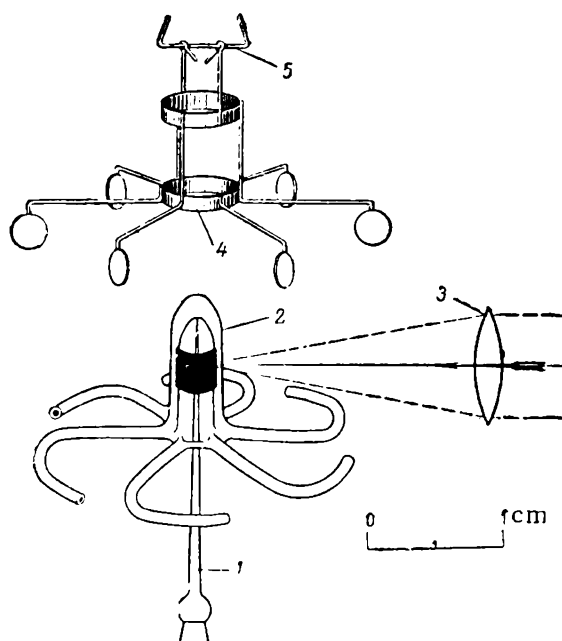

Fig. 18. Device for studying the reaction of the jet ("spider") which operates on the principle of a Segner's wheel.

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The same conclusion is reached from an analysis of the results of the following ingenious experiment performed by Kapitza [4]. On the point of a needle 1 (Figure 18) is placed a small double-walled flask 2 which is placed on the needle with the "dome" side up. The space between the walls communicates with the external helium by means of six bent capillaries which form a device something like a Segner wheel. The helium inside this "spider" is heated by means of a point-source lamp which is focussed by lens 3. The inner walls are blackened so that the energy which is absorbed is relatively large. When the lamp is turned on the system rotates with velocities up to 120 rpm. With all other conditions being the same the rate of rotation increases as the temperature is lowered. When the fitting marked 4, consisting of two rings and six vanes soldered to it, is placed on the spider by means of suspension 5 rotation of the spider ceases. Under these conditions each of the vanes is 1 - 2 mm from the corresponding capillary exit. Thus, the reactive force of the jet is responsible for the rotation of the spider.
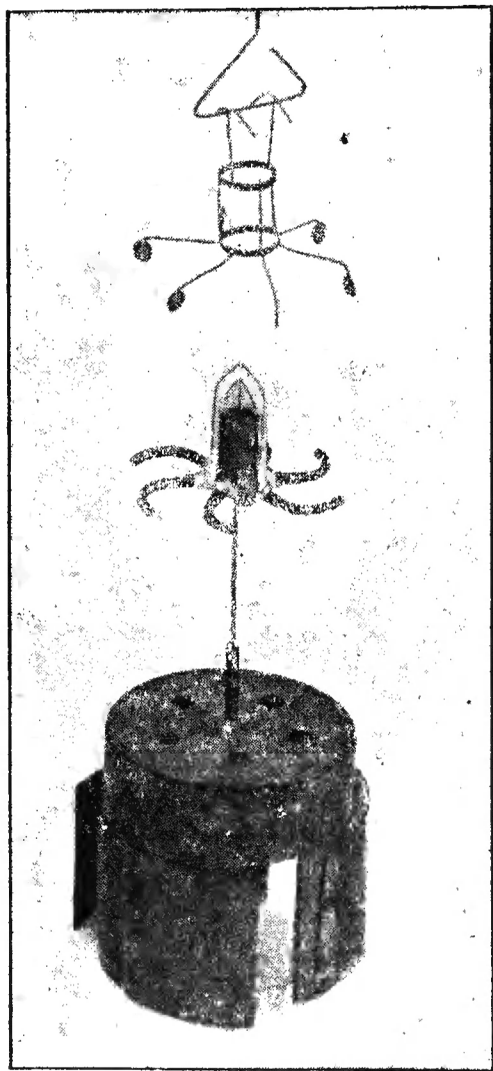
Fig. 19. Photograph of the spider.

A photograph of the spider is shown in Figure 19. The temperature dependence of the jet pressure on the vane (circles) and the reactive pressure of the jet on the flask (points) are shown in Figure 20. The experimental points lie close to a line which intersects the temperature axis at a point not far from the λ-point.

HEAT TRANSPORT IN FREE HELIUM II. RADIOMETER EFFECT. An investigation of the heat transport mechanism in free

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helium II is of considerable interest since one does not know *a priori* that the behavior of helium in a capillary and in bulk are the same. Investigations of this nature were of especially great value during the time in which a quantum theory of superfluidity had still not been formulated.

Fig. 20. Temperature dependence of the jet pressure at an applied power of 0.7 watts/cm²:
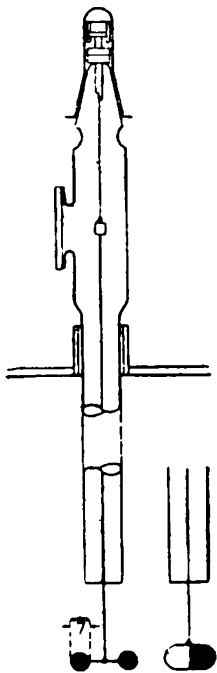
Fig. 21. Strelkov radiometer.
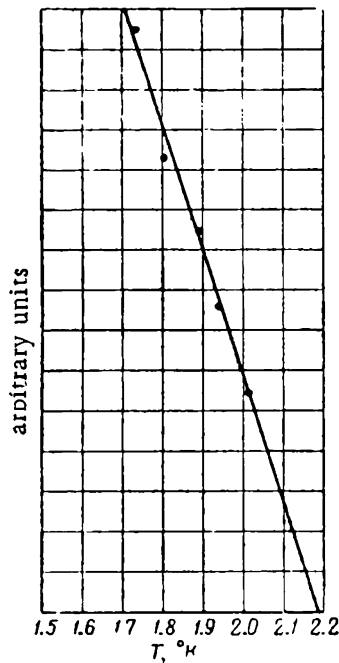
Fig. 22. Temperature dependence of the radiometer effect.

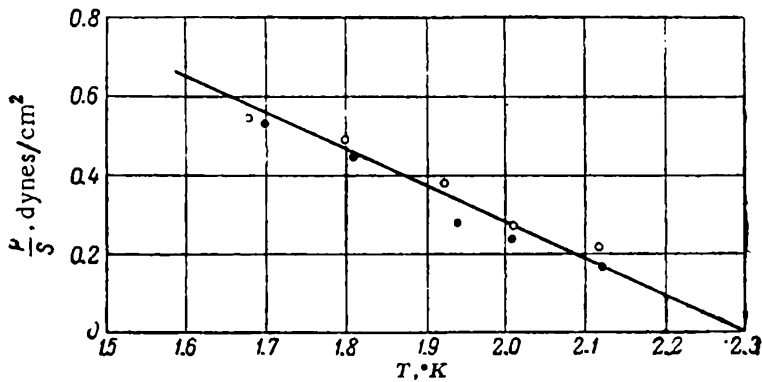In Section 6 of Chapter VI a description was given of a device by means of which the radiometer effect was investigated by Strelkov. The Strelkov radiometer [27] is shown in Figure 21. As has been

indicated in Section 6 of Chapter VI, at temperatures above the λ-point a radiometer submerged in helium I "interacts" with the light beam in the same way as all other liquids: it is "attracted" toward the source of light. On the other hand, a helium II radiometer is "repelled" by the light source, indicating the existence of strong convection currents which have nothing in common with the ordinary convection currents of gravitational origin.

The effect becomes stronger as the temperature is reduced. The relation is more or less linear (cf. Figure 22); within the limits of the experimental errors, this result is in agreement with the results obtained by Kapitza [4] for the pressure exerted by a jet on a vane. According to the Strelkov data, at high heat loads the angle of rotation of the radiometer increases in linear fashion with the load. As the heat load is reduced the linear relation gives way to one which is approximately quadratic (Figure 23).
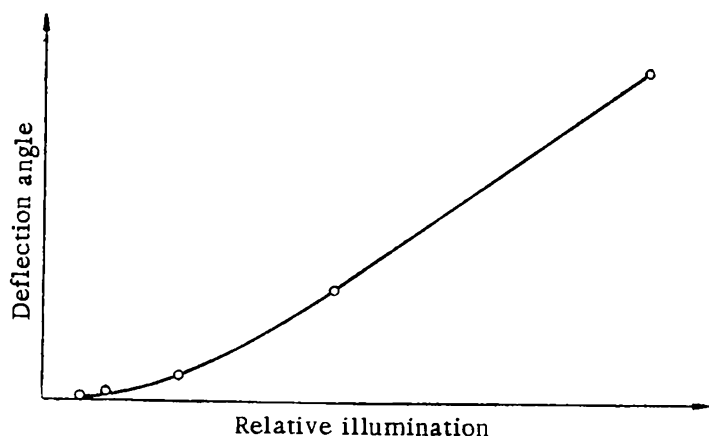
Fig. 23. Radiometer effect as a function of applied power.

DISCUSSION OF THE RESULTS. HEAT TRANSPORT MECHANISM IN HELIUM II. The theory of thermal quanta given in Chapter VIII was formulated on the basis of the experiments described in the preceding section. From the point of view of this theory the effect studied by Kapitza can be interpreted as follows. At the surface of the heat source enclosed by the vessel thermal quanta — rotons and phonons — are produced. Thus, there is an increased concentration of thermal quanta inside the vessel. Because of the concentration gradient the gas of thermal excitations streams along the capillary, forming a sort of thermal wind.

Because they have an effective mass, the rotons and phonons carry a momentum which is easily measured by means of the vane and torsional weights used by Kapitza. The appreciable pressure of the jet and the fact that it maintains its shape show that at low temperatures heat which propagates in the form of thermal quanta is characterized by inertia.

In the foregoing we have used the term "gas of thermal excitations." The theory indicates that far from the λ-point, the ensemble of rotons and, especially, phonons can actually be considered an ideal gas, consisting of noninteracting quasi-particles. At constant pressure the number of quasi-particles per unit volume of this gas is a sharp function of temperature. Since the current of thermal excitations issues from the capillary under the effect of a difference in concentrations, the above implies the existence of a temperature difference, which, in the majority of cases, is so small that it can hardly be measured even when the pressure of the jet on the vane is appreciable.

Since the transport of heat in helium II is realized through the flow of thermal excitations, it is not surprising that any disturbance of this flow involves a reduction in heat transport. On this basis the effect on heat transport of such things as pressure variation, agitation, flow along a capillary, etc., becomes completely clear. In most cases in which the heat transport is disturbed by the application of high heat loads (these effects are discussed in Sections 5 and 8 of the present chapter) any further effect due to one of the mechanical factors, for example, flow (cf. Figure 8), becomes less important.

Observations of the radiometer effect in free helium II indicate that the heat transport in this case is undoubtedly of the same nature as that in thin capillaries. Moreover, in this case the heat, which has inertial properties, is carried in helium II by thermal quanta which exert a reactive force on the surface close to which they are produced.

A number of effects, such as the dependence of the jet pressure and divergence on temperature, the way in which the pressure of a jet increases with increasing load, superheating in helium II and so on will be analyzed in detail in subsequent sections.

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Section 2. The two Forms of Motion in Helium II

FORMULATION OF THE PROBLEM. In the theory of super-fluidity formulated by Landau it is assumed that two forms of motion are possible in helium II. One form is the simple motion of an ordinary viscous liquid while the other form corresponds to the potential flow of an ideal liquid which exhibits no friction effects.

In the second form of motion, which is called "superfluid" motion, in particular the relation curl $\mathbf{v}$ = 0 obtains.

Each of these forms of motion may be characterized by its own density: the normal by the density ρ_n and the superfluid by the density ρ_s. These motions can be excited separately or simultaneously so that the total momentum of helium II is

$$\mathbf{j} = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n \tag{9.1}$$

($\mathbf{v}_s$ and $\mathbf{v}_n$ are the velocities associated with the superfluid and normal motions). The sum of the densities ρ_s and ρ_n is equal to the ordinary density of helium II which, in general, is more or less independent of temperature: $\rho_s + \rho_n = \rho$. In particular, forms of motion are possible in which there is no net transfer of mass and for which the helium II, as a whole, remains at rest. This motion is described by the relation

$$\mathbf{j} = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n = 0. \tag{9.1a}$$

From these considerations it follows that, among other things, when a body is moved in helium II only part of the liquid is set in motion, in particular, the normal part. The other part of the liquid, which is associated with the superfluid form of motion, is not carried along by the displacement of the body. Thus, for example, if an axially symmetric container is rotated in helium II, not all of the mass enclosed by the container takes part in the motion, as would be the case for ordinary liquids; only the normal component rotates. The remaining part of the liquid filling the rotating container remains at rest.

These effects, which seem paradoxical at first, were verified in experiments carried out by Andronikashvili [6].

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DESCRIPTION OF THE EXPERIMENT. In the Andronikashvili experiments the rotation of a container is replaced by the torsional oscillations of a stack of parallel metal disks fastened to a common axis (Figure 24); the distance between neighboring disks, (on the basis of the depth of penetration of a viscous wave characteristic of a given mode of oscillation) is chosen to ensure the most complete entrainment of the liquid by the rotating parts of the system. The stack is enclosed by a metal container which is rigidly connected to it; the system is suspended by a glass rod clamped to a thin phosphor-bronze wire which hangs from the roof of a helium Dewar (cf. Figure 25).
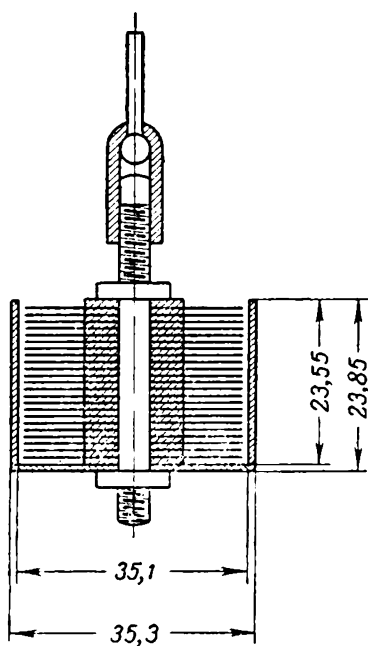
Fig. 24. Diagram of a device used for measuring the density of the normal component.
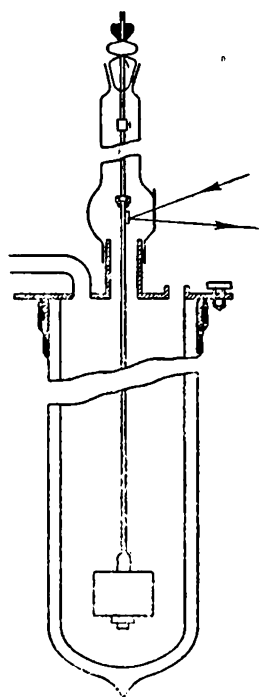
Fig. 25. Apparatus for measuring the density of the normal component.

The experiment consists of measuring the period of oscillation (and thus the moment of inertia) of the suspended system; this quantity depends on the density of the liquid which takes part in the oscillatory motion of the system. It is obvious that those motions of the system or its elements which lead to a simple agitation of the liquid are avoided.

The extremely small density of liquid helium and the extremely small viscosity, even for helium I, impose exceptionally stringent requirements on the construction of the apparatus, which is literally a skeleton. Thus, the disks which entrain the liquid are made from aluminum foil 0.0013 cm thick while the distance between them is 0.021 cm. On hundred disks are used.

RESULTS OF THE EXPERIMENT. The experiment verified the hypotheses which had been made above. In Figure 26 is shown the temperature dependence of the oscillation period for the system submerged in liquid helium, referred to the oscillation period at the λ-point, which is taken as the unit of comparison. The behavior of the curve above the λ-point corresponds to an increase in the density of helium I as the temperature is varied from the boiling point to the λ-transition. Below the λ-point, as has been noted above, the normal density of helium II remains essentially constant. However, as is apparent from Figure 26, in this temperature region there is a very marked change in the moment of inertia of the system.

It follows from a consideration of the hydrodynamics of viscous liquids that under the conditions of the experiment described above the density of the liquid which takes part in the motion of the system can be determined from the expression

$$\rho_n = \frac{2I(\theta^2 - \theta_0^2)}{\pi N a \theta_0^2 R^4} \frac{1}{1 + \frac{\delta}{aN}\left(1 + \frac{2L}{R}\right)}, \tag{9.2}$$

which represents an approximate solution to the Navier-Stokes equation with appropriate boundary conditions. In equation (9.2) I denotes the moment of inertia of the system, N is the number of disks, a is the distance between disks, R is the radius of the cylindrical container which surrounds them, L is the length of the generating lines, δ is the depth of penetration of the viscous wave, and θ_0 and θ are the oscillation periods for the system when empty and when filled with liquid helium. It should be noted that the second term represents a correction for the entrainment of the liquid by the outer surfaces. It is reasonable that the mass of this layer is larger, the greater the viscosity of the medium.

In order to reduce the error in the measurement, instead of determining the absolute value the relative value of ρ_n is determined by comparison with the density ρ_λ of liquid helium 1 at the temperature of the λ-point. Then in the first approximation, in the
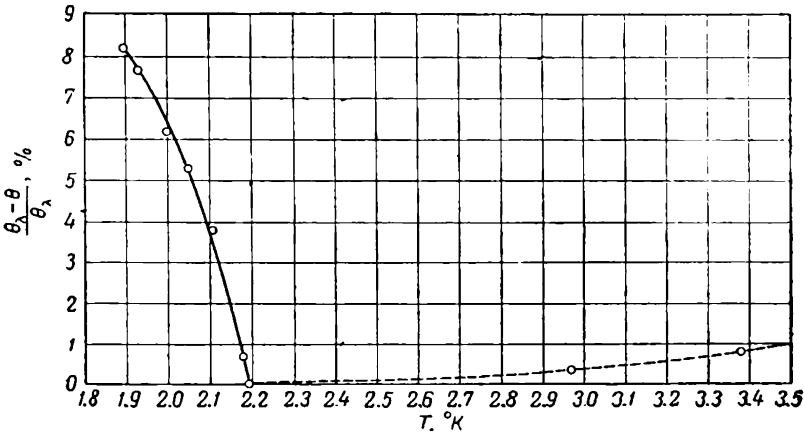
Fig. 26. Temperature dependence of the oscillation period of a stack of discs used for measuring the density of the normal component of helium II.

formula for ρ_n/ρ_λ there appear only the oscillation periods; the constants of the system appear only in a correction factor

$$\frac{\rho_n}{\rho_\lambda} = \frac{\theta^2 - \theta_0^2}{\theta_\lambda^2 - \theta_0^2} \cdot \frac{1 + \frac{\delta_\lambda}{aN}\left(1 + \frac{2L}{R}\right)}{1 + \frac{\delta}{aN}\left(1 + \frac{2L}{R}\right)} \tag{9.3}$$

The temperature dependence of the density of the liquid which takes part in the motion of the system is shown in Figure 27. The data used to plot this curve were obtained with $\theta_\lambda = 4.42$ sec and $\theta_0 = 3.77$ sec [7].

As is to be expected, the density of helium 1 determined by this method is found to be close to its true value.

Subsequent measurements of the normal density of helium II, carried out by Andronikashvili [8] are shown in Figure 28 and in Table 1.

The same quantity can be determined from measurements of the velocity of propagation of second sound (cf. Section 8 of the

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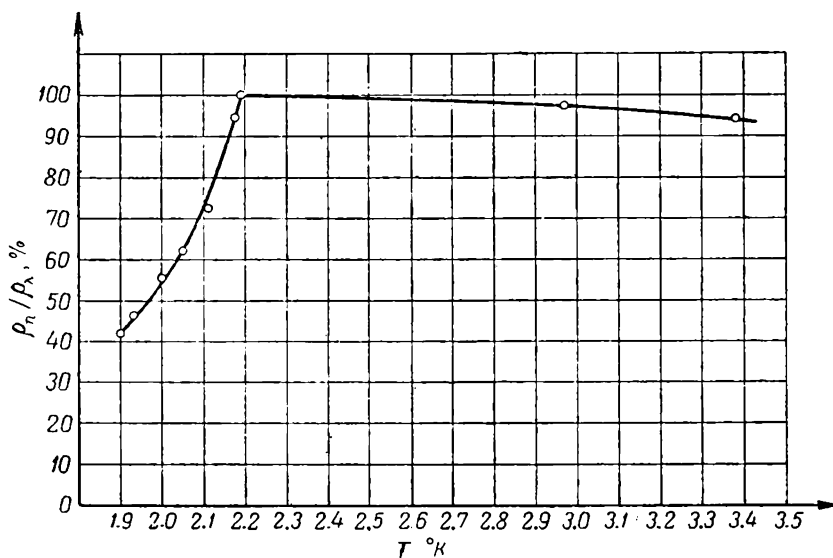
Fig. 27. Temperature dependence of the density of He I and the density of the normal component of He II.

present chapter), carried out by Peshkov [9]. The data obtained by Peshkov are shown in Figure 28 (points) and in Table 2.

TABLE 1

Temperature Dependence of the Density of the Normal Component ρ_n after Andronikashvili
(percent of the density ρ_λ of helium I)

$T,°K$	2.21	2.143	2.113	2.073	2.03	1.995	1.94
ρ_n/ρ_λ, %	100	85.6	78.2	70.4	62.5	57.6	49.5
$T,°K$	1.890	1.830	1.731	1.640	1.529	1.427	1.334
ρ_n/ρ_λ, %	44.7	36.1	28.0	19.2	11.8	7.0	4.3

Within the experimental errors of both experiments the measured values of ρ_n/ρ_λ agree over the entire temperature region which has been investigated. However, the error in the determination of this quantity as obtained in the Andronikashvili experiments increases sharply as the temperature is reduced (from 2% at the λ-point to 17% at $T = 1.34°$ K) whereas the error in the Peshkov experiment remains constant at about 5 - 6%. Hence, in the low temperature region (from 1.6 - 1.7° K) it is more desirable to use the Peshkov data.

TABLE 2

Temperature Dependence of the Density of the Normal Component ρ_n after Peshkov.
(in percent of the density ρ_λ of helium I)

$T, °K$	2.15	2.1	2.05	2.0	1.95	1.9
ρ_n/ρ_λ, %	90.1	78.5	68.0	59.3	51.8	45.0

$T, °K$	1.8	1.7	1.6	1.5	1.4	1.3
ρ_n/ρ_λ, %	33.6	25.2	18.3	11.9	7.6	4.7

DISCUSSION OF THE RESULTS. The interpretation which has been offered of the results of the experiment described above is open to a very natural objection. The point is that effects identical with those described can occur by virtue of effects other than a change in the density of the liquid which oscillates with the system. A reduction in the oscillation period of the system could just as well be due to a reduction in the viscosity of the liquid on transition through the λ-point. It is true that the penetration depth, which determines the amount of liquid entrained by the device, does depend on the viscosity η:

Fig. 28. Temperature dependence of the density of the normal component: ◯ - after Andronikashvili, ● - after Peshkov.

$$\delta = \sqrt{2\eta/\rho\omega}. \qquad (9.4)$$

However, as is apparent from equation (9.4), the penetration depth also depends on the oscillation frequency ω. Thus, if the entrainment of the liquid by the oscillating disks is not complete, the amount of liquid taking part in the motion of the system should depend on frequency. The fact that the quantity ρ_n/ρ_λ is independent of period in the range from 4 to 32 seconds indicates that in these experiments the entire mass of liquid between the disks which is capable of rotation is, in fact, entrained.

It may also be noted that in all cases it is found that the logarithmic decrement is independent of amplitude, indicating the absence of such effects as turbulence or critical flow. The maximum peripheral velocity, reached at θ = 4.42 sec, is 0.6 cm/sec.

Thus, on the basis of the experiments described here we have direct evidence of the existence of two forms of motion in helium II: "normal" and "superfluid." In addition, the present method turns out to be capable of measuring the effective densities to be assigned to each of these forms of motion. These effective densities are found to be exponential functions of temperature.

The hydrodynamics of helium II, developed by Landau (cf. Chapter VIII), which leads to the notion of two forms of motion in helium II, makes possible an analysis of a whole series of experimental results which had not been understood earlier. In general, the solution of these problems is facilitated by the fact that the hydrodynamic equations for all processes in helium II can be broken down into two independent equations — one for the normal part and the other for the superfluid.

Section 3. Viscosity of the Normal Component

THE NOTION OF VISCOSITY IN HELIUM II. In Section 2 of Chapter VI two methods have been described for the measurement of viscosity: a method involving the damping of oscillations of a disk or cylinder, and a method based on flow through a capillary. We now consider the first of these. As is well known, when a solid body is submerged in a viscous liquid and moved, a layer of the liquid in contact with the solid surface is entrained by it and is displaced with the same velocity. At infinity the liquid is at rest. Under these conditions the spatial velocity distribution is governed by the Navier-Stokes equation and is determined by the boundary conditions for the particular case. If the solid body executes oscillations in an infinite liquid, the distance at which the velocity of the liquid falls off by a factor of e is called the penetration depth. As has already been noted the penetration depth is a function of the viscosity of the liquid and is related to it by the expression

$$\delta = \sqrt{2\eta/\rho\omega},$$

where δ is the penetration depth, η is the viscosity of the liquid, ρ is the density and ω is the angular frequency of the oscillation.

However, since liquid helium II is a "mixture" of two liquids it becomes important to decide to which liquid should be assigned the notion of a penetration depth: the liquid in the ordinary sense of the word or the normal part alone. Physically, both of these cases are by no means the same, as is apparent from the following example. Let us assume that the surface of a solid body submerged in helium II oscillates with respect to another fixed solid surface and that the distance between them is comparable with the depth of penetration. If we assume that the viscous properties are a characteristic of the entire liquid, in equation (9.4) we must use the ordinary density of helium II, thus obtaining some value for δ. Assuming that the viscous properties should be assigned only to the normal component, using the latter density we would obtain another value for the depth of penetration δ_n which would be larger than the first. Thus, if the distance between the mobile and fixed surfaces is smaller than δ_n but larger than δ according to the first point of view the mobile surface would be oscillating in an infinite liquid; if the second point of view is adopted it should be "stuck" to the fixed surface. It is clear that the boundary conditions which are applied in solving the Navier-Stokes equation depend on which point of view is adopted.

It will be shown below that the experimental results lead to the conclusion that it is physically meaningful to assign a viscosity to the normal component only — not to the liquid as a whole.

CRITIQUE OF THE EARLIER EXPERIMENTS. This important factor was not taken into account by earlier investigators. Thus, Keesom and MacWood [10], in measuring the viscosity at 1.3° K and assigning it to the liquid as a whole overestimated the value by approximately a factor of 20 (at these temperatures the density of the normal part is about 5%). Since the density also appears in the expression for the depth of penetration, in computing δ the error was 2,000%. It is obvious that the boundary conditions chosen by these authors are highly unreliable, especially if note is taken of the fact that they observed damping of a disk enclosed in a cylinder with very little space between them. Hence it is no surprise that the data obtained by these authors does not yield the correct value of the density of the normal component.

The Keesom and MacWood experiments preceded work by Wilhelm, Misener and Clark [11] who determined the viscosity from the damping of oscillations of a cylinder with two-sided conical fittings suspended in an infinite liquid. The unusual temperature behavior of the viscosity of helium I and its high absolute values make the work of these investigators highly unreliable. Because these authors did not publish their experimental data in detail, it is difficult to determine the possible sources of error.

DETERMINATION OF VISCOSITY FROM EXPERIMENTS IN WHICH TWO FORMS OF MOTION ARE OBSERVED. A general solution of the problem of the oscillations of a system of parallel disks in liquid helium II leads to two equations with two unknowns. One of these equations

$$\pi N \nu \rho_n R^4 \frac{\omega}{\delta} \left[\frac{\sin \frac{a}{\delta} + \mathrm{sh}\, \frac{a}{\delta}}{\cos \frac{a}{\delta} + \mathrm{ch}\, \frac{a}{\delta}} + \frac{1}{N} \left(1 + \frac{2L}{R} \right) \right] = I\, (\omega_0^2 - \omega^2) \tag{9.5}$$

has already been used in Section 2 in determining the density of the normal component. [In equation (9.5) ν denotes the kinematic viscosity.] The other equation yields a value for the penetration depth, from which the normal viscosity is easily determined:

$$\frac{\mathrm{sh}\, \frac{a}{\delta}}{\cos \frac{a}{\delta} + \mathrm{ch}\, \frac{a}{\delta}} =$$

$$= \frac{\omega_0^2 - \omega^2 + 2\omega\, (\gamma - \gamma_0)}{\omega_0^2 - \omega^2 - 2\omega\, (\gamma - \gamma_0)} \cdot \frac{\sin \frac{a}{\delta}}{\cos \frac{a}{\delta} + \mathrm{ch}\, \frac{a}{\delta}} - \frac{1}{N} \left(1 + \frac{2L}{R} \right). \tag{9.6}$$

Here γ_0 and γ denote the damping of the system in vacuum and in liquid helium, respectively. (The significance of the other symbols is the same as in Section 2.) Equation (9.6) is solved graphically. Its sensitivity to errors in the experimentally determined quantities means that the value of δ_n is very approximate; consequently the value of η_n is also approximate. In the formula for ρ_n the penetration depth appears only in the correction term so that the inaccuracy of the method has little effect on the final results. However, this situation can be highly inconvenient if one is interested in an exact determination of the normal viscosity. An analysis of the temperature dependence of η_n yields all the data required for the

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qualitative conclusions [8] which lie at the basis of the later investigation of the viscosity of the normal component.

It turns out that the viscosity of helium II falls off rapidly as the temperature is reduced below the λ-point (approximately the same as ρ_n) down to 1.9 - 1.85° K. Starting at these temperatures the viscosity remains constant over a certain range; at still lower temperatures, in the region of 1.5° K, an increase is noted.

This statement is based on the results of later experiments.

DETERMINATION OF THE VISCOSITY OF THE NORMAL COMPONENT FROM EXPERIMENTS WITH HEAVY DISKS. A more exact determination of the viscosity of the normal component can be obtained using disks or cylinders which are so heavy that one can neglect the temperature variation in the density of the entrained liquid. Experiments of this kind were carried out by Andronikashvili [7], [12] who used a system of copper disks 0.5 mm thick and 35.1 mm in diameter. The disks were suspended on an axis with the following spacings between them: 1) 25 disks at 0.8 mm, 2) 15 disks at 2 mm, 3) 10 disks at 3 mm. There was also a single disk, which oscillated in an "infinite" liquid.

The viscosity was determined from the simplified expression

$$\eta_n = \frac{4I^2(\gamma - \gamma_0)^2\theta}{\pi^5 N^2 R^5}\frac{1}{\left(1 + \frac{2d}{R} + \frac{2\delta}{R}\right)^2},\tag{9.7}$$

the first factor of which is the solution of the Navier-Stokes equation for a system of disks of radius R oscillating about their normal; the second term represents an edge correction. Since the correction term contains the penetration depth, a successive-approximation method was used to compute the viscosity. In equation (9.7) I is the moment of inertia of the disk system, γ is the damping in helium, γ_0 is the damping in vacuum, θ is the oscillation period, N is the number of disks, and d is the thickness of an individual disk.

Equation (9.7) is derived under the assumption that the distances between the disks are large compared with the penetration depth. If this assumption holds true the values of the viscosity determined from the damping of each of the four systems should be the same.

The experiment shows, however, that at temperatures far from the λ-point the viscosity computed with the simplified formula, using the damping of the first system (with a distance of 0.8 mm between disks), is considerably smaller than that of the three remaining systems. This means that the simplified formula should not be used in this case since the layer of liquid enclosed between the disks oscillates as a whole without splitting up into different velocity components. This effect would not occur if the actual depth of penetration depended on the total density of helium II rather than the density of its normal component; as was noted above, the second quantity is always larger than the first.

In the present sub-section we present the temperature dependence of the viscosity, obtained with a system (cf. Figure 29) consisting of a single brass disk a (diameter 39.44 mm and thickness 1.0 mm), a straight glass rod b, connected with the disk by means of a threaded coupling c, a steel clamp d, with a mirror and a filament of phosphur bronze e, 0.07 mm in diameter, which is the elastic suspension.
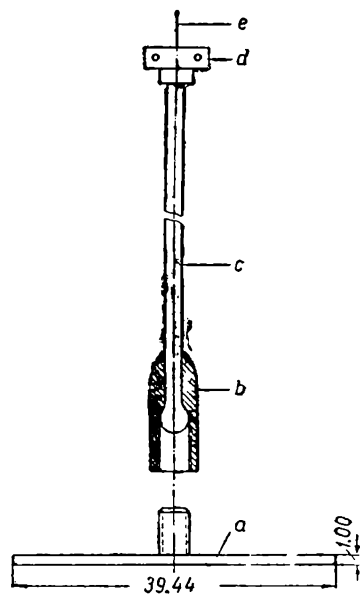
Fig. 29. Device for measuring viscosity.

TABLE 3

Temperature Dependence of the Viscosity of the Normal Component η_n.

T, °K	2.168	2.130	2.073	2.003	1.943	1.858	1.802	1.745
$\eta_n \cdot 10^{-5}$ poise	2.05	1.75	1.45	1.28	1.18	1.12	1.13	1.125
T, °K	1.690	1.621	1.560	1.502	1.425	1.391	1.357	1.320
$\eta_n \cdot 10^{-5}$ poise	1.125	1.12	1.19	1 33	1.58	1.76	1.89	2.04

It is apparent from Table 3 and Figure 30 that the normal viscosity falls off steeply between the λ-point and 1.9° K. In the temperature region 1.6−1.9° K it remains constant; thereafter, it starts to rise sharply.

DETERMINATION OF THE VISCOSITY OF THE NORMAL COMPONENT FROM EXPERIMENTS ON HEAT TRANSPORT IN SLITS. As will be shown in Section 6, there is a linear relation between the heat flow, the magnitude of the thermo-mechanical effect and the difference of temperature (in the region of thermal loads below the critical value); the factor of proportionality in this relation contains the viscosity of the normal component. The observations were made by Meliink [13] whose experimental results were analyzed by Andronikashvili [14].
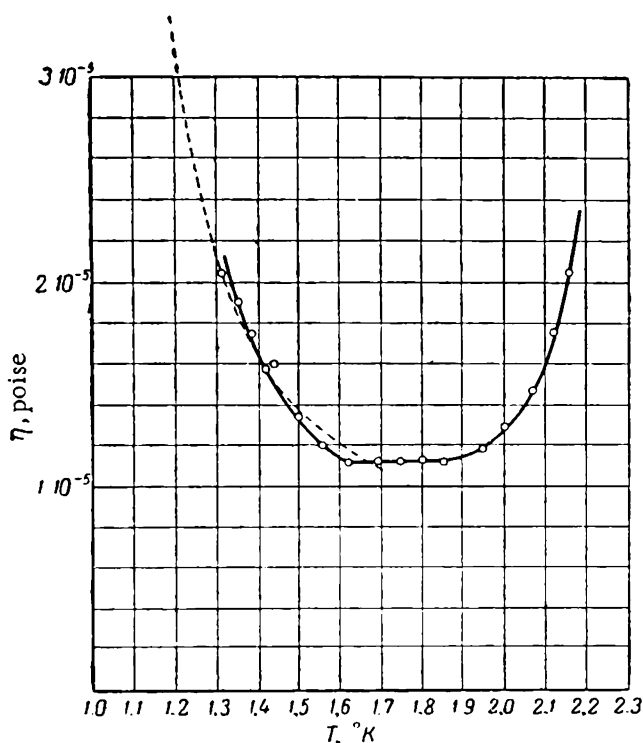
Fig. 30. Temperature dependence of the viscosity of the normal component from oscillating-disk measurements.

According to these data, the viscosity of the normal component for the temperature region between 1.7 and 1.9° K is $1.71 \cdot 10^{-5}$ poise for a slit width of 10.5 μ and $1.18 \cdot 10^{-5}$ poise for a slit width of 5 μ. The latter value is in good agreement with the values obtained by Andronikashvili [12] (cf. preceding sub-section). The discrepancy for the slit width of 10.5 μ may be attributed to the large error with which the width of the slit could be measured by Mellink.

The most important feature, however, is the agreement between the temperature behavior of the viscosity observed by Andronikashvili, and that obtained from an analysis of the data obtained by Mellink. In Figure 31 are shown the values of viscosity taken with the value at $T = 1.7°$ K as a reference. The solid curve refers to the Andronikashvili heavy-disk experiments while the circles and dots refer to slit widths of 10.5 and 5μ respectively.
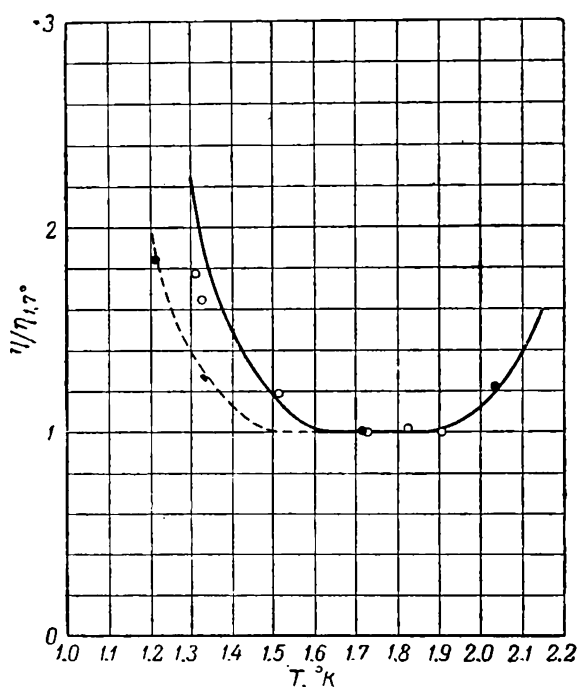
Fig. 31. Temperature dependence of the viscosity of the normal component as obtained from the Andronikashvili measurements (solid curve) and computed from the Mellink data (◯ slit 10.5 μ ; ● slit 5μ).

It is worthy of note that at temperatures below 1.5° the values of the apparent viscosity for the narrower slits are below the solid curve. This effect — a reduction in the apparent viscosity, which sets in as the slit width is reduced — becomes very marked for slit widths between 1 and 0.15 μ (cf. Section 6).

DISCUSSION OF THE RESULTS. As has already been indicated, transport effects in helium II are due to a gas of thermal quanta which at low temperatures may be considered an ideal gas.

Simplifying the very complicated processes associated with transport effects, the unusual temperature behavior of the viscosity can be explained qualitatively as follows.

At temperatures far from 0° K the number of phonons is considerably smaller than the number of rotons. Thus, it is reasonable to assume that the basic feature in the transport of angular momentum is scattering of rotons on rotons. Now, the concentration of rotons is a sensitive function of temperature. In an ordinary gas the viscosity is independent of pressure; similarly, in the first approximation the viscosity of the roton gas should be independent of the temperature since the product of the number of particles and their free path in the roton gas remains fixed at all temperatures. And, indeed, in one temperature region the viscosity is found to be completely independent of temperature (the flat part of the curve). The temperature region close to the λ-point cannot be included in these considerations because in this region the concentration of rotons becomes so large that the roton concept loses significance.

Using the formula developed in the Landau theory and the experimental data on viscosity Andronikashvili was able to determine the effective roton-roton scattering cross section; at $T = 1.7°$ K this quantity is found to be $2.2 \cdot 10^{-14}$ cm^2, i.e., approximately the same as the scattering cross section of the helium atom.

The low-temperature branch of the curve can be explained by the fact that phonons take part in transport effects at temperatures below 1.5° K. A consideration of this aspect of the problem led Landau and Khalatnikov [15] to the conclusion that the phonon free path length, for collisions with rotons, increases rapidly as the temperature is reduced and that this is the reason for the sharp increase in viscosity (the data of these authors are shown in Figure 30 by the dashed line).

As the temperature is reduced further, so that the free path length of the phonons becomes comparable with the dimensions of the container (rarified gas), the viscosity of helium II should reach a maximum. The magnitude and position of this maximum will depend on the actual experimental conditions. The departure of the viscosity values for thin slits from the true value can be explained by the long path length (Figure 31). Owing to the fact that the phonon path length becomes comparable with the width of the slit at low

temperatures, when the gas of thermal quanta flows through such thin channels, the well-known effect of slipping should be observed; this effect arises because the velocity of the gas does not vanish at the surface of the solid body, and consequently there is a momentum discontinuity at the walls. Thus, the deviation of the points from the solid curve not only supports the presently held ideas concerning the phonon-roton gas but, indeed, serves as one of the strongest arguments for this interpretation.

Thus, from an analysis of the experiments on the determination of the viscosity of the normal component we are led to the conclusion that it is composed of two parts: a roton part and a phonon part; on the other hand, the effects due to both parts are inseparable in all experiments presently known to us.

Section 4. Reversibility of Hydro-Thermal Processes and the Thermo-Mechanical Effect

HEAT CONTENT OF HELIUM II. The experiments described in Sections 1 and 2 establish uniquely the general nature of the two countercurrents by means of which heat is transported in helium II. It is apparent that if we try to explain heat transport in helium II by means of this particular picture, we must grant that both currents are in fundementally different energy states. This formulation of the problem was proposed by Kapitza, who solved it experimentally in a very simple and ingenious way [16].

At the basis of the method lies an effect which has been noted by a number of authors and which consists of the following. A container which communicates with a helium bath via wall film or capillary and which is filled with helium II is observed to become cooler (so-called mechano-caloric effect). Under these conditions a temperature difference is produced which hinders the further flow of the liquid and an equilibrium state is established quickly. However, if heat is generated inside the container it becomes filled with helium very quickly, and in this way, it is possible to obtain considerable differences in levels (the so-called thermomechanical effect). By generating an amount of power in the container such that no temperature difference arises, knowing the volume of the liquid which flows through the capillary it is possible to determine the difference between the specific heat content of the liquid in the capillary and the heat content in the volume.

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In the Kapitza experiment the capillary was replaced by an adjustable narrow slit formed of two ground quartz slabs *a* and *b* (cf. Figure 32). One of these slabs, serving as a flange for the vessel **1**, has a central aperture through which the helium flows into the
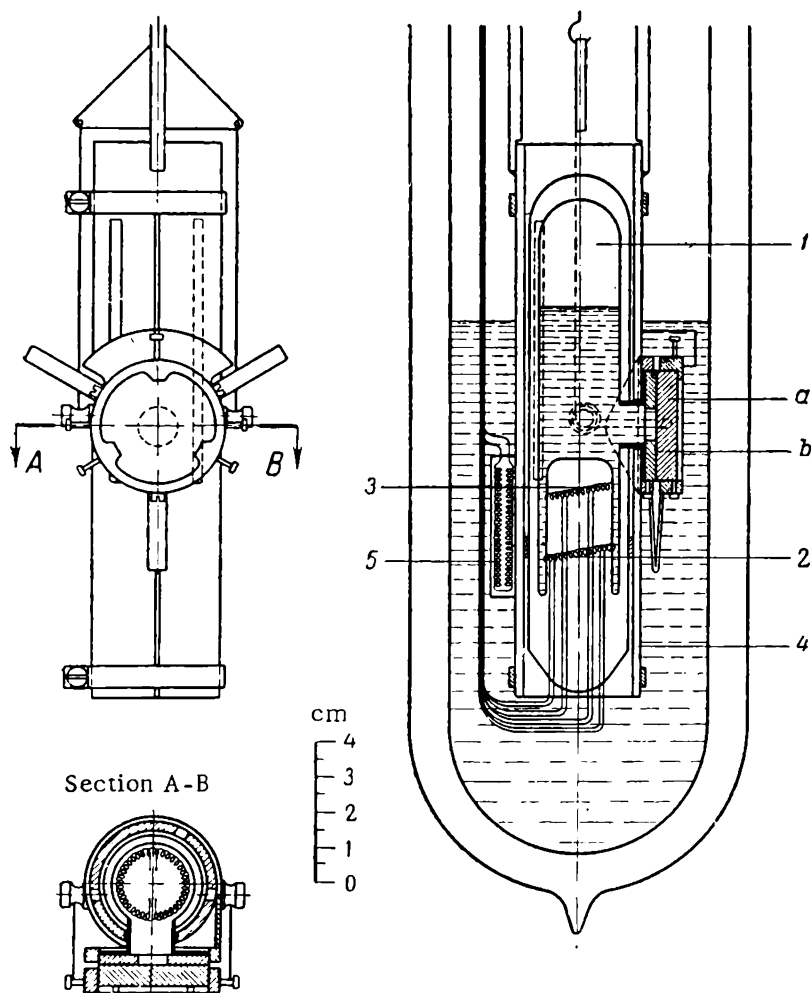
Fig. 32. Device for studying the hydro-thermal properties of helium II.

inside of the system. Inside the vessel there is a constantan heater 2 and a phosphor-bronze thermometer 3; outside, there is a housing and a copper radiation shield 4 with narrow longitudinal slits to provide visual observation. There is a mechanism by means of which the width of the slit can be changed during the course of the experiment. The internal thermometer 3 is balanced against an external thermometer 5 of the same resistance.

Because of warping when the device is cooled, the slit between the quartz plates is wedge-shaped. At highest compression of the disks the width is taken as $1.3 \cdot 10^{-5}$ cm.

During the course of the experiment observations are made of the bulk flow rate v (the velocity of displacement of the meniscus inside the bulb) as a function of the applied power q. Simultaneously, measurements are made of the difference in temperatures in the helium inside the vessel and in the bath. At the start of the experiment the liquid levels in both containers are made the same so that the difference in hydrostatic pressures can be neglected.

As is apparent from Figure 33, at small values of q, the bulk flow rate increases in proportion to the heat load as shown on the figure by the solid line. This section of the curve corresponds to the absence of a temperature difference between thermometers 3 and 5 (Figure 32). At a certain load the bulk flow rate starts to fall off and a difference in temperature is observed (shown on the curve by dots). There is little doubt that at this point the linear velocity of flow through the slit reaches the critical value beyond which motion of the helium is accompanied by losses.

The difference in specific heat content of the flowing helium and free helium is determined by the slope of the line $v = f(q)$ and at $\Delta T = 0$ is

$$Q = \frac{1}{\rho} \frac{dq}{dv} \text{ cal/g} \qquad (9.8)$$

where ρ is the density of the liquid helium.

Fig. 33. Bulk flow rate and temperature difference as functions of applied power. Curve 1 refers to a slit $d = 0.3$ μ; curve 2 refers to a slit $d = 3.0\,\mu$.

The values of the difference in heat content obtained in this manner are denoted by the symbol ◑ in Figure 34. However, it should be noted that Kapitza did not make certain corrections which should be applied in equation (9.8); these arise in connection with the heat of condensation of the vapor. Condensation occurs because of the reduction of the volume of the container filled with liquid. Thus, the true values of the difference in heat content are somewhat higher: at the λ-point the correction is approximately 5%, and at 1.3° K it is approximately 9%.

THERMO-MECHANICAL EFFECT AND REVERSIBILITY OF HYDRO-THERMAL PROCESSES. The experiment described in the preceding sub-section shows that helium which flows through a narrow slit is undoubtedly in an energy state which differs from the state of the free helium. This situation as well as the origin of the difference of levels due to the generation of heat at one side of the slit led Kapitza to the idea of using the reverse effect for determining the difference in heat content: using pressure to force helium II through a slit it is possible to produce a difference in temperature on both sides of the slit (the mechano-caloric effect). This experiment is described in the work which has been cited [16] (cf. a discussion of this effect in Section 6).

Fig. 34. Temperature dependence of the heat content of helium II.

The vessel (cf. Figure 32) is submerged at a level considerably below the level of the helium in the cryostat. The experiment is carried out by measuring the difference of temperature under these conditions as a function of the difference in levels h of the liquid in the vessel and in the bath. Figure 35 shows that the temperature difference depends on the level difference in linear fashion, and that the effect increases monotonically as the temperature is reduced.

Fig. 35. Thermo-mechanical effect in narrow slits.

Starting from the laws of thermodynamics it can be shown that for a reversible process the following relation should hold between the temperature and pressure differentials:

$$p = ApQ\frac{\Delta T}{T},\qquad\qquad (9.9)$$

where A is the mechanical equivalent of heat, and $p = \rho g h + \left(\frac{\partial p}{\partial T}\right)_T \Delta T$ is the resultant pressure, made up of the pressure of the column of liquid of height h and density ρ and the difference in the vapor pressures above the surfaces, one of which is at temperature T and the other, at temperature $T + \Delta T$. Thus, the state of minimum potential energy corresponds to equilibrium for the resultant of the hydrostatic and "thermal" pressures in the liquid.

If the assumption is valid, that the flow of helium through a slit is reversible, the difference in the heat content Q determined from equation (9.8) should agree with the value obtained from equation (9.9).

The difference in heat content, determined from the static experiments described in the present paragraph, obtained for different temperatures and slit widths are shown in Table 4 and Figure 34 (circles).

TABLE 4

Heat content Q of Helium II as a Function of Temperature at Different Slit Widths d.

(Kapitza)

T, °K	2.105	2.04	1.965	1.880	1.740	1.580	1.470	1.345	1.33
Q, cal/g	0.67	0.535	0.43	0.33	0.205	0.107	0.064	0.035	0.032
d, μ	0.14	0.14	3.2	3.2	3.2	0.14	3.2	3.2	0.25

As is apparent from the curve, within the accuracy of the experimental errors, which include the correction for the heat of condensation of the vapor (necessary in the dynamic experiments), the flow of helium through a slit is thermodynamically reversible. In this same figure, for purposes of comparison are shown the values obtained from the Landau theory (dots).

The reversibility of the hydrothermal processes observed by Kapitza is possible only if the entropy of the helium which flows through the slit is zero. Thus the quantity Q, measured in the

Kapitza experiments, represents the specific heat content of free helium II.

It should be especially emphasized that the flow of helium along a capillary, slit, film, etc., is, in all cases, controlled by the total pressure $\bar{\pi}$ made up of the pressure p and the temperature pressure $A\rho Q\frac{\Delta T}{T}$ so that

$$\pi = \rho g h + \left(\frac{\partial p}{\partial T}\right)_T \Delta T + A\rho Q\frac{\Delta T}{T}. \tag{9.10}$$

NEW METHOD OF OBTAINING LOW TEMPERATURES. The considerable reduction of temperature which takes place at relatively weak pressures, of the order of 100 mm, of a helium column (corresponding to approximately 1 mm Hg), and the monotonic increase in this effect as the temperature is reduced have been used as the basis for a newly developed method of obtaining low temperatures; this scheme was proposed by Kapitza [16]. In his experiments the helium II was forced through porous filters by large pressures. The observed temperature difference was approximately 0.4° per single stage. There is little doubt that a still greater temperature reduction can be achieved by sequential use of such a system.

This method would seem to be free of the theoretical limitations which characterize other methods of obtaining low temperatures.

THERMO-MECHANICAL EFFECT AND THE VISCOSITY OF THE SUPERFLUID COMPONENT. Above, we have given an equation (9.10) which indicates that the total pressure which is effective in the flow of helium II through a narrow slit is made up of a hydrodynamic component and a temperature component. This conclusion, first reached by Kapitza, is of fundamental importance in analyzing a whole series of effects which occur in helium II. In particular, it must be taken into account in analyzing all experiments relating to the determination of viscosity in which capillary viscosimeters are used. Indeed, at points where the helium moves in a narrow slit or capillary an amount of heat is generated which is sufficient to produce a counterpressure which can hinder the flow of the liquid along the capillary.

Analyzing the original experiments carried out by Kapitza [2] (cf. Chapter VI), it may be maintained that heat is generated at the inner perimeter of the circular slit. Hence, the actual effective pressure is lower and it follows that the viscosity value obtained by Kapitza — 10^{-9} poise — is too high. The same applies to the experiments carried out by Allen and Misener [3].

In the experiments of Giauque, Stout, and Barieau [17], described in detail by Keesom in Chapter VI, although the lower end of the slit was submerged in the helium bath, owing to which it obviously should have been at the temperature of the bath, at the upper end of the slit, where the helium vapors were pumped off, heat was absorbed and the temperature pressure undoubtedly acted to oppose the flow. Thus, in the experiments reported by these authors the value of the viscosity must be considerably higher than the true value. It is obvious that the conditions in which the helium flow took place were very far from the isothermal conditions assumed.

It should be noted that the correct value of the viscosity of the superfluid component can be obtained only if one observes the flow of helium through a slit which is so narrow that the motion of the superfluid component is not accompanied by motion of the normal component. If the capillary is relatively wide, as was the case in the experiments of Jones, Wilhelm and Grayson-Smith (cf. Section 2, Chapter VI), the flow of the liquid is determined basically by the viscosity of the normal component, particularly at high temperatures where the percentage content of the normal component is relatively high. Hence, it would not be surprising that these authors obtained values of the viscosity close to the value obtained in the oscillating disk experiments. However, because of the non-uniformity in the cross section of the capillaries and the absence of any corrections for end effects the viscosity coefficient in these experiments is still two or three times larger than might be expected.

Taking account of the thermo-mechanical effect Kapitza, on the basis of his own experiments described in the work cited [16], determined the upper limit of the viscosity for the superfluid component. Under the assumption of laminar flow he obtained a viscosity value of $\eta \approx 10^{-11}$ poise.

Meyer and Mellink [18] quote a value for η of the order of 10^{-12} poise.

DISCUSSION OF THE RESULTS. As has been indicated in Chapter VII, the thermo-mechanical effect can be easily understood if one invokes an analogy with the osmotic pressure which operates in the case of a semi-permeable membrane. In the present case the capillary or slit takes the place of the membrane while the role of the molecules in solution is played by the thermal excitation quanta and the role of the solution, by the superfluid helium.

Here we may point out two possibilities for investigating the thermo-mechanical effect. One of these — the dynamic approach — is realized in the Kapitza experiments. Heat flowing into a thermally isolated container which communicates with a helium bath only through the narrow slit tends to increase the concentration of thermal quanta in the container. The tendency toward equilibration of concentrations in these experiments is realized exclusively by the flow of the superfluid part, which is characterized by zero entropy. However, it is important that the applied power be small enough so that no temperature difference arises between both sides of the slit. Under these conditions all of the applied heat is consumed in motion of the superfluid mass of helium, which is deficient in heat content. The generated power should be small so that the velocity of the superfluid component does not exceed its critical value because the transition through the critical velocity means a temperature difference. In turn, a temperature difference can force thermal excitations, through even a narrow slit; as a consequence, it becomes impossible to obtain unique results from the experiment.

Another possibility in investigation the thermo-mechanical effect lies in observing the physical quantities in the stationary state. In this case, because the amount of helium inside the reservoir is fixed, the generation of heat can involve only an increase in the temperature difference, i.e., an increase in the concentration of thermal excitations, or, as it is generally termed, the temperature pressure. This temperature pressure must be equalized by the hydrostatic pressure of the column of liquid helium. Under stationary conditions the equilibration of the concentration of thermal excitations on both sides of the slit results not only from the flow of the superfluid part of helium II, but, as has been indicated above, from the fact that the normal part of the fluid it forced through the slit.

Thus, the investigation of the thermo-mechanical effect under stationary conditions is related in a direct way to the investigation of heat transport in helium II which fills narrow slits. This aspect of the thermo-mechanical effect will be considered in Section 6.

An analogy with osmosis is also completely applicable in explaining the method of obtaining low temperatures proposed by Kapitza. In this case one deals with the filtration of thermal excitations by means of a system of narrow capillaries. The superfluid mass which passes through the capillary will, to a large extent, be free of thermal excitations; this effect becomes more pronounced as the capillary becomes narrower. In the limit, for infinitely thin capillaries through which only the superfluid component can pass, the temperature of the "filtrate" should be zero.

Section 5. Critical Velocities

CRITICAL VELOCITIES AND WALL FILMS. The motion of helium II in a wall film is completely understandable in terms of the theory of thermal motion in a quantum liquid if the film is considered a very narrow capillary. In such a capillary the thermal excitations, experiencing friction at the walls, remain fixed although the number of excitations in the film is by no means zero and is determined by the temperature of the wall (the latter differs slightly from the temperature of the liquid which forms the film). Thus, only the superfluid part of the helium II can move along the film. In a certain sense, the presence of the normal mass in the film leads to a reduction in the cross section of the film; this reduction becomes greater as the temperature of the liquid comes closer to the λ-point. Hence, the true velocity with which the superfluid part moves along the film turns out to be ρ/ρ_s times greater than the quantity which has been taken by various authors in the motion of the film and used in their computations of the liquid flow.

Since the film may be considered an analogue to a narrow capillary one would expect to find thermo-mechanical and mechano-caloric effects at its "ends." In this connection we may make mention of a very curious fact established by Strelkov [19]: namely, the velocity of the film is independent of a difference in levels if this difference is greater than 0.5 mm. This level difference corresponds to a pressure differential of 6.5 dynes/cm.

At small pressure differentials the film velocity is also affected by the temperature pressure, which is of equal importance with the hydrostatic pressure. If the hydrostatic pressure works against the temperature pressure, regardless of the absolute hydrostatic pressure the film velocity is controlled by it only when it exceeds the temperature pressure by a sizeable margin. In the absence of a temperature pressure the velocity of the film would be independent of the pressure differential starting at zero level difference.

The fact that there are velocities which are independent of the pressure differential indicates flow at the limiting (critical) velocity.

To determine the critical velocity of the superfluid component the data given in Section 5 of Chapter VI are multiplied by the ratio ρ/ρ_s. When this is done the temperature dependence of the critical velocity is given by the values in Table 5.

TABLE 5

Critical Velocity v_c in a Film as a Function of Temperature
[converted for superfluid density]
(computed from the experimental data of Daunt
and Mendelssohn [42])

T, °K . . .	2.03	1.93	1.91	1.90	1.83	1.81	1.75	1.695	1.65	1.5	1 3
$v_c^{(s)}$, cm/sec	12	15	13	19	20	26	23	22	23	23	21

Although the thickness of the film and its temperature dependence have not been investigated in great detail, the data in Table 5 may be taken as evidence of the fact that the critical velocity of the film does not change with temperature.

CRITICAL VELOCITIES IN THIN SLITS AND CAPILLARIES. a) *Kapitza experiments.* In the experiments carried out by Kapitza [16] the flow of the superfluid component through a narrow slit was a result of heat generated inside a thermally insulated vessel. As has already been noted in Section 4, at small thermal loads the influx takes place under conditions for which it is impossible to detect any temperature difference between the vessel and the helium

cryostat: it is less than 10^{-4} degrees. When the generated power is increased to some critical value, a temperature difference suddenly appears and the bulk flow rate is reduced considerably.

The linear velocity determined from the volume flow of liquid through the slit corresponding to the increased temperature difference was taken as the critical velocity by Kapitza. In Table 6 presented below are shown the critical velocities corresponding to various temperature differences and different slit widths d. The third line contains the velocity values computed by Kapitza while the fourth line contains the true velocities of the superfluid component. As has been indicated by the author the slit was wedge-shaped; thus the possibility is not excluded that the values of the critical velocity are too large by a considerable margin.

TABLE 6

Critical Velocities for Slits of Different
Widths at Various Temperatures

T, °K . . .	1.64	1.64	1.935	1.557
d, μ	3	0.3	0.14	0.14
v_c, cm/sec	14	40	80	110
$v_c^{(s)}$, cm/sec	18	51	153	131

As is evident from Table 6, for a given temperature the critical velocity increases as the slit width is reduced. A change in temperature produces only a small variation in the critical velocity.

b) *Experiments of Meyer and Mellink.* Observing the dependence of the thermo-mechanical effect on the difference in temperature between two chambers separated by a thin slit, Meyer and Mellink [18] found that up to some definite flow velocity the level of the liquid corresponded to the increase in the temperature difference, without any noticeable relaxation effects. If, however, the temperature difference was increased rapidly, the fountain effect could not follow it because of a number of retarding effects. The flow velocity corresponding to the appearance of relaxation effects was taken as the critical velocity. In Table 7 are shown the values of the critical velocity at various temperatures. In the opinion of these authors the errors in

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these values are smaller than 20%. As in the preceding table, the third column contains the values of the critical velocity computed under the assumption that all of the helium participates in the flow and the fourth column gives the true value of the velocity of the superfluid component with respect to the walls of the slit and the fixed normal mass.

TABLE 7

Critical Velocity in a Thin Slit as a Function of Slit Width and Temperature (after Meyer and Mellink)
[converted for superfluid density]

slit width, μ	T, °K	crit. vel. v_c, cm/sec (not converted)	crit. vel. $v_c^{(s)}$, cm/sec
1	1.66	15	19
1	1.78	12	18
1	1.84	16	26
1	1.95	16	33
1	2.00	>12	>29
1	2.16	< 3	<33
0.2	1.34	>22	>23
0.2	1.58	23	28
0.2	1.89	18	32
0.2	1.99	20	46
0.2	2.15	< 3	<24
0.3	1.38	>16	>17
0.3	1.45	20	22
0.3	1.52	20	23
0.3	1.68	>11	>14
0.3	1.82	> 7	>16
0.3	1.94	13	25
0.3	2.09	<16	<59
0.3	2.12	6	30
0.3	2.16	3	33
0.3	2.17	< 2	<33
0.15	2.176	< 1.8	<35

It is apparent from Table 7 that if the critical velocity does depend on temperature the dependence is very weak. No dependence of critical velocity on slit width was observed in these experiments.

c) *Experiments of Allen and Misener*. Critical velocities can also be obtained from the experiments of Allen and Misener [20], who

carried out a detailed investigation of the flow of helium II through capillaries of different widths and lengths.

These experiments, which were carried out under very complicated conditions, can be interpreted quantitatively only for the cases in which the bulk flow of the normal mass can be neglected, i.e., when the flow takes place through very narrow capillaries.

As has been noted in Section 2 of Chapter VI the thinnest capillaries were obtained by drawing a metal tube filled with a bundle of steel wires. In the flow of helium through a system of such capillaries the flow velocity was found to be independent of the pressure differential between 5 and 160 dynes/cm^2. The fact that the velocity is independent of pressure, an effect which is completely unknown for classical liquids, can be explained only by the existence of a critical flow.

These considerations are illustrated in Table 8: the flow velocity is shown as a function of temperature and pressure differential for a system of capillaries with mean diameter of $1.2 \cdot 10^{-5}$ cm.

TABLE 8

Flow Velocity of Helium II Through a Capillary of Width $1.2 \cdot 10^{-5}$ cm as a Function of Temperature and Pressure Differential
(computed from the experimental data of Allen and Misener).

T, °K	p, dynes/cm^2	v_c, cm/sec	$v_c^{(s)}$, cm/sec
1.2	15	12 9	13.5
1.2	160	19.9	13.5
2.16	15	1.0	10
2.16	160	1.25	12.5

The temperature dependence of the critical velocity is shown in Table 9. The data refer to the same system of capillaries and were obtained with a pressure differential of 160 dynes/cm^2.

Within the limits of the experimental errors it may be assumed that the critical velocities computed for the superfluid mass are indeed temperature independent.

CRITICAL VELOCITIES IN WIDE SLITS. In all the experiments considered in the previous subsections the value of the critical velocity was determined from the volume of the liquid flowing through a capillary, slit, or film per unit time. Under these conditions the hydrostatic and temperature pressures are so small that the normal component can be considered immobile. Thus, in all these experiments it is the velocity of the superfluid component with respect to the walls which is considered.

TABLE 9

Temperature Dependence of the Critical Velocity for $d = 1.2 \cdot 10^{-5}$ cm at a Pressure Differential of 160 dynes/cm^2
(computed from the experimental data of Allen and Misener).

T, °K . . .	2.17	2.165	2.153	2.13	2.102	2.07
v_c, cm/sec	0.73	1.07	1.62	2.36	3.30	4.45
$v_c^{(s)}$, cm/sec	12.0	13 4	14.7	13 1	13.7	14.6
T, °K . . .	2.035	1.90	1.65	1.38	1.31	1.20
v_c, cm/sec	4.95	7.8	11.6	12.9	12.6	12.9
$v_c^{(s)}$, cm/sec	13.3	13.9	14.9	13.9	13.3	?

In wide slits and at high pressures the normal mass can no longer be considered immobile and critical flow is determined by the velocity associated with the relative motion of both components.

In these cases the observation is generally carried out under stationary conditions with the level difference in the liquid at both sides of the slit unchanged. Under these conditions the critical velocity cannot be computed from the liquid flow rate and must be determined from the heat consumption. Actually, because of the peculiar mechanism for heat transport in helium II, the mean velocity of the normal component at some given temperature is

completely determined by the specific applied power φ and the specific heat content Q:

$$\dot{\overline{v}}_n = \frac{\varphi}{\rho Q},\qquad\qquad(9.11)$$

and, since no transfer of mass occurs in the stationary conditions, the velocity of the superfluid component v_s is determined from equation (9.1a). Obviously, to compute the critical velocity it is more convenient to determine v_s not from the mean velocity of the normal mass, but from its maximum value, which exceeds the mean velocity v_n by a factor of 1.5 in the case of a plane slit.

If one analyzes the experimental data obtained by Mellink [13], indicating the dependence of the thermo-mechanical effect on the flow of heat (this is considered in detail in Section 6) in a plane slit of width 10.5 μ, the temperature dependence of the critical velocity which is obtained is shown in Table 10.

TABLE 10

Temperature Dependence of the Critical Velocity
for a slit width of 10.5 μ
(computed from the experimental data
obtained by Mellink)

T, °K . . .	1.32	1.51	1.725	1.83
v_c, cm/sec	$59 < v_c$	$19.6 < v_c < 27$	10.5	$v_c < 10.3$

T, °K . . .	1.90	1.96	2.05	2.15
v_c, cm/sec	$7.5 < v_c$	$7.6 < v_c$	$v_c < 7$	$v_c \ll 9$

Because of the poor accuracy in the measurements of the differences in temperature and pressure and because of the absence of any sharp break in the curve showing the dependence of the thermo-mechanical effect on heat flow, there is a considerable spread in the values for the temperature dependence of the critical velocity. Nevertheless, the data in Table 10 gave the first indication of changes in the critical velocity with changes in the absolute temperature.

DISCUSSION OF THE RESULTS. As has already been noted in Chapter VIII the theory of thermal quanta indicates the need for a critical velocity at which the motion of the superfluid component is no longer reversible and is accompanied by thermal effects. However, on the basis of the theory one would expect a disruption in the superfluid flow at velocities of the order of several tens of meters per second whereas the actually observed critical velocities are smaller than several tens of centimeters per second.

At the present time no explanation has been forthcoming for the observed break in the superfluid flow; whether or not it is due to a mechanism which can be predicted by the theory is still not known.

In spite of the fact that in almost all experiments (with narrow gaps) the critical velocity apparently remains fixed (although the width of the gap varies from $3-5 \cdot 10^{-6}$ for films to 10^{-4} cm for slits, i.e., by a factor of 20) there is still some basis for expecting that the value of the critical velocity should be a function of the characteristic dimensions of the problem. However, it is impossible to ascertain the exact nature of this dependence on the basis of the presently available experimental data. Certain authors propose, for example, that the critical velocity should become smaller as the dimensions of the container are increased, falling off as $1/\sqrt{r}$. Other authors indicate an inverse proportionality in which the critical velocity obeys the relation

$$v_c = 0.79 \cdot 10^{-4} \frac{1}{r} \quad \text{cm/sec.} \tag{9.12}$$

An examination of Table 7 shows that with a slit width of 10^{-4} cm the mean value of the critical velocity is 26 cm/sec. For a container with characteristic dimensions of $2 \cdot 10^{-2}$ cm we should find a critical velocity of the order of $1.5-2$ cm/sec according to the first point of view and $0.15-0.2$ cm/sec according to the second.

Using the fact that the logarithmic decrement is completely independent of amplitude, [established in observations of the oscillations of a stack of disks with a period of four seconds (cf. Section 2)] it may be asserted that the critical velocity for a situation with characteristic dimensions of $2 \cdot 10^{-2}$ cm is, in any case, greater than 0.6 cm/sec. Thus it can be shown that a dependence of the form a/r for the critical velocity as the function of container dimensions is incorrect to begin with.

At the basis of equation (9.12), obtained from theoretical considerations by de Boer, Milelson and Bijl, lies the incorrect notion that helium II is a mixture of two types of atoms, one of which is in a degenerate state while the other occupies energy levels different from zero (excited state). Hence it is not surprising that the application of quantum-mechanical considerations to this incorrect model leads to relations which do not agree with reality. It should be noted, however, that the point of view being discussed here has received unjustified acceptance in many countries.

Up to the present time a functional relation of the form $1/\sqrt{r}$ has not been checked over a sufficiently wide range of characteristic dimensions; on the other hand, the difference in the mean values of the critical velocities for slits of $1\,\mu$ and $10.5\,\mu$ is generally suggestive of this type of relation.

Section 6. Heat Transport in Slits and Capillaries

GENERAL REMARKS. Until very recently the capacity of helium II to transport heat was characterized by the quantity k defined by the classical relation

$$k = \frac{\varphi}{dT/dx},$$ (9.13)

in which k is the "classical thermal conductivity," φ is the density of heat flow and dT/dx is the temperature gradient.

However, as is well known, the "classical thermal conductivity" is found to be a function of load. This result was first obtained by Allen, Peierls and Uddin [5] and subsequently verified by Kapitza [4] who found that in the region of very low temperatures its value is a value ten times greater than that found earlier by Keesom, reaching 10^5 watts/deg. cm.

The main results of numerous investigations of the heat transport process in helium II (cf. Section 3, Chapter VI) may be summarized as follows: the heat flow is proportional to the square root of the temperature difference; with a fixed temperature difference the heat flow as a function of temperature passes through a maximum in a region of $1.9°$ K; with capillary diameters which are fractions

of a millimeter the heat flow is proportional to the cross section and inversely proportional to the length; the heat flow is proportional to the fountain effect regardless of the applied temperature difference.

Certain deviations from these empirical rules are observed at low temperatures (about 1.2° K) in capillaries whose diameter is of the order of 10 μ. Under these conditions Allen and Reekie [21] have been able to establish the fact that the heat flow is proportional to the temperature difference.

It should be noted that in certain individual cases some investigators have found departures from the empirical relations listed above. For example, Kapitza [4] established that the thermal conductivity increases as the capillary cross section is reduced. The essence of this experiment (Figure 6) is as follows. A vessel, which is essentially a Dewar flask, is connected to a helium bath by means of a capillary. Inside the bulb there are a heater and a bronze thermometer. When the heater is switched on the temperature inside the bulb increases. Knowing the heat flow, the difference in temperature between the vessel and the bath, and the geometric dimensions of the capillary, it is possible to compute the "thermal conductivity" using the formula given above (9.13). By introducing a fine glass filament into the capillary and thus reducing its cross section, at a fixed heater load it is possible to reduce considerably the pressure differential at the ends of the capillary, indicating that the "thermal conductivity" increases under these conditions.

From what has been presented it is clear that the problem of heat transport in helium II is extremely complicated and that the experimental observations are frequently inconsistent with each other.

A detailed and systematic investigation of heat transport and its connection with thermo-mechanical effects has been carried out by Keesom and his school [13], [18], [22].

APPARATUS. These investigations were carried out using apparatus constructed by Keesom and Duyckaerts [22]. The main part of the system consists of the flask F and the housing H (cf. Figure 36). The flask is a thin glass capillary D surrounded by a vacuum jacket. The bottom part of the capillary is flared, forming part of a

concave spherical surface a. In the housing H, also surrounded by the vacuum jacket, there is a heater C and a bronze thermometer B. The upper part of the housing forms a convex spherical surface b. The surfaces a and b are ground and carefully polished to ensure complete optical contact. In this way a thin slit is formed through which the helium can flow into the inner volume of the system. The slit is circular. The diameter is approximately 2.5 cm. The flask is attached to the housing by three helical springs. The slit gap can be regulated between 0.1 and 19 μ, as desired, by means of thin spacers (tungsten wires, silk fibers and fine ink marks). The length of the slit is not the same in the different experiments and varies from 1 to 2.75 mm.

The slit width is determined in preliminary experiments from the flow velocity of a gas of known viscosity. These measurements are checked by a Newton's ring method.

When assembled, the housing and flask are placed in the outer chamber and filled with liquid helium by condensation. As mentioned above, the inner cavity of the system is connected with the outer chamber through the slit but is otherwise thermally insulated from it. On the other hand the outer chamber is in best possible thermal contact with the cryostat although separated from it by solid walls. The temperature in the outer chamber is measured with the bronze thermometer A which is similar to the thermometer B located in the inner chamber.

WIDE SLITS. The experiments of Keesom and Duyckaerts [22] were devoted to the fountain effect (thermo-mechanical effect) and heat transport in relatively wide slits. Most of
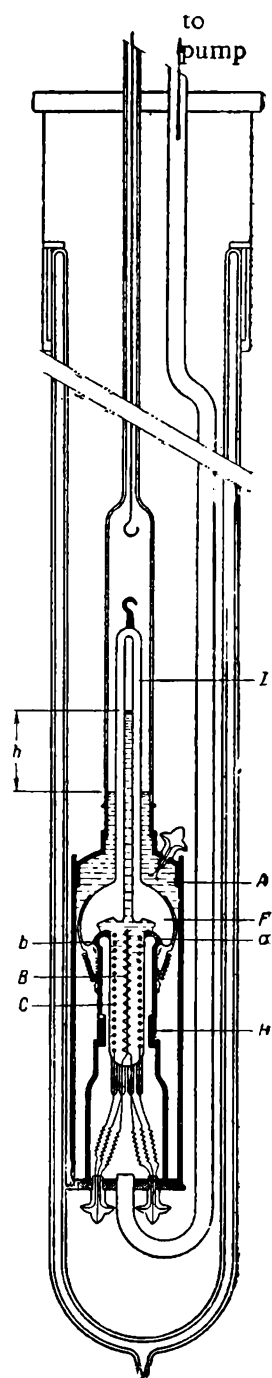

Fig. 36. Device for studying heat transport and the thermo-mechanical effect.

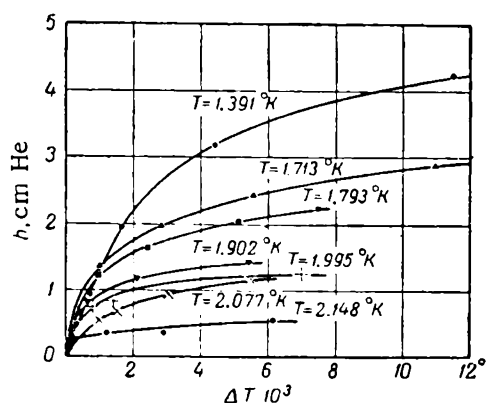their results refer to slits 9.3 and 19 μ wide and 2.48 mm long. Because of certain limitations in their apparatus the measurements could not be carried out at slit widths less than 2 μ because under these conditions there was enough weak illumination from the neon light for the level of the helium in the capillary to reach its peak. The difference in temperatures produced under these conditions was so small that measurements could not be carried out.

Fig. 37. Relation between the fountain effect and temperature difference. Slit 19μ.

According to the data of Allen and Reekie [21], for temperature differences less than 0.001° the fountain effect increases with increasing temperature, reaching a maximum between 1.39 and 1.71°, and then starts to fall off as the temperature is increased further. At greater temperature differentials the effect falls off monotonically right up to the λ-point, starting at the lowest pressures obtainable. The relation between the fountain effect, expressed in centimeters of a column of liquid helium, and the temperature difference on both sides of the slit 19 μ in width is shown in Figure 37.

Wide slits have been studied in greater detail by Mellink [13], using the same apparatus as the above investigators. His experiments refer to slits of width 10.5 and 5 μ and length 2.48 and 1.0 mm, respectively.

According to Mellink, the position of the maximum on the curve showing the temperature dependence of the fountain effect for wide slits is determined not only by the slit width but also by the temperature difference. When this temperature difference is increased the maximum is displaced toward lower temperatures (cf. Figure 38). An increase in slit width displaces the maximum in the same direction. The theoretical values for the fountain effect are shown by dashed lines in the figure. The agreement between the theoretical and experimental values extends over a larger region when the temperature difference is smaller and the slit is narrower.

We may note (as follows from the remarks in the paper by

Keesom and Duyckaerts) that the thermo-mechanical effect as taken by these authors is the difference in pressure produced by the difference in levels. The additional pressure difference due to the difference in vapor pressures, brought about by the different temperatures on both sides of the slit, was not taken into account in the thermo-mechanical effect as defined above. However, equation (9.9) contains the total pressure difference which, as has been shown by Kapitza, consists of both terms indicated above.

Keesom and Duyckaerts used slit widths of 0.75, 1.5−2, 1.15, 9.3 and 19 μ in studying heat transport. The slit length was 2.75 mm in the first three slits and 2.48 mm in the others. For purposes of comparison an experiment was also carried out using a cylindrical capillary 0.0947 mm in diameter which was mounted in the system used by Keesom, Saris and Meyer [23].

Fig. 38. Temperature dependence of the fountain effect for various temperature gradients. Slit 5μ.

In measuring the heat flow all losses due to the incomplete thermal isolation were taken into account.

The method of observation was as follows: power was applied to the heater located in the inner cavity of the system. Using two bronze thermometers, one on each side of the slit, the temperature difference between the inner and outer chambers was ascertained. Measurements of the thermometer resistance were carried out every ten seconds before heating, during heating, until equilibrium was achieved, and after heating. The difference in helium levels on both sides of the slit was determined with a cathometer.

Similar experiments were carried out by Mellink [13] using slits of 10.5 and 5 μ. In contrast with the method used by Keesom and Duyckaerts, the temperature difference in these experiments was measured with two thermometers used in a balanced bridge circuit. With this arrangement it was possible to avoid errors due to
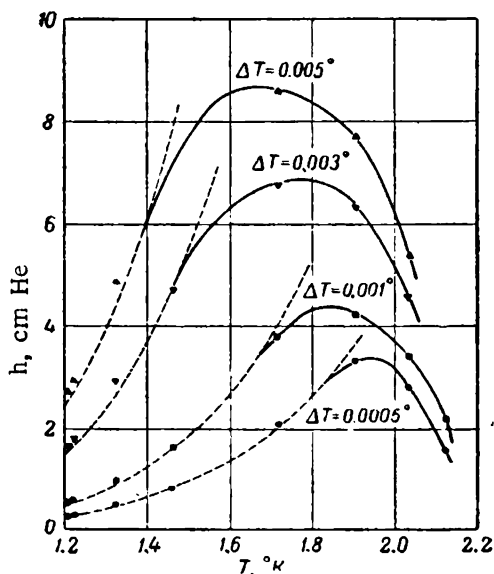
temperature variations in the bath in successive measurements of the resistance of the inner and outer chambers. The measurements were carried out under stationary conditions as described in Section 4.
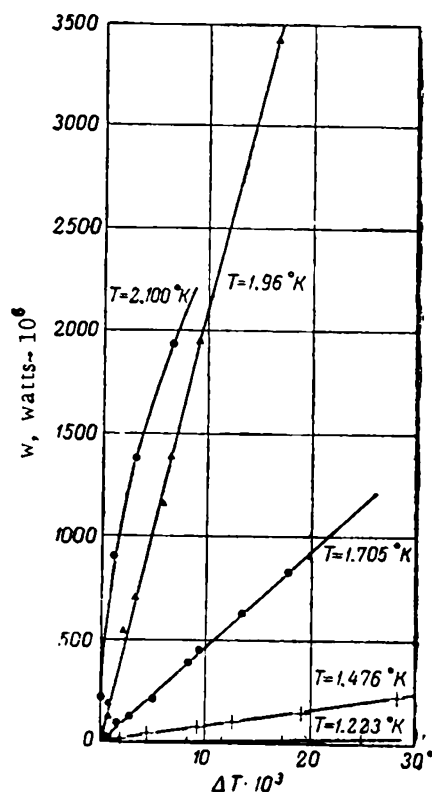
Fig. 39. Dependence of heat flow on temperature difference. Slit $1.5 - 5\,\mu$.
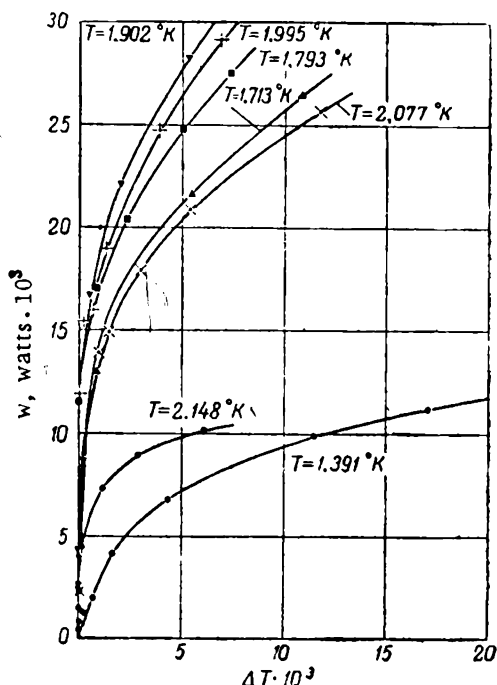
Fig. 40. Dependence of heat flow on temperature difference. Slit $19\,\mu$.

The results may be summarized as follows. For relatively thin slits (of those which were investigated) the heat flow is proportional to the difference in temperatures between the inner and outer chambers for a fixed value of the bath temperature up to temperatures close to the λ-point (cf. Figure 39). As the slit width is increased the region in which this linear relation obtains gradually decreases, remaining only at relatively low temperatures. The wider the slit, the lower the temperature at which this relation applies. For a slit width of $19\,\mu$ the dependence of heat flow on temperature difference, as shown in Figure 40, does not differ very much from the dependence exhibited by relatively wide cylindrical capillaries observed in earlier work.

For slits of 5 and 10 μ the linear dependence of flow on temperature gradient is found only at low temperature values. It vanishes completely in the immediate vicinity of the λ-point (Figure 41).
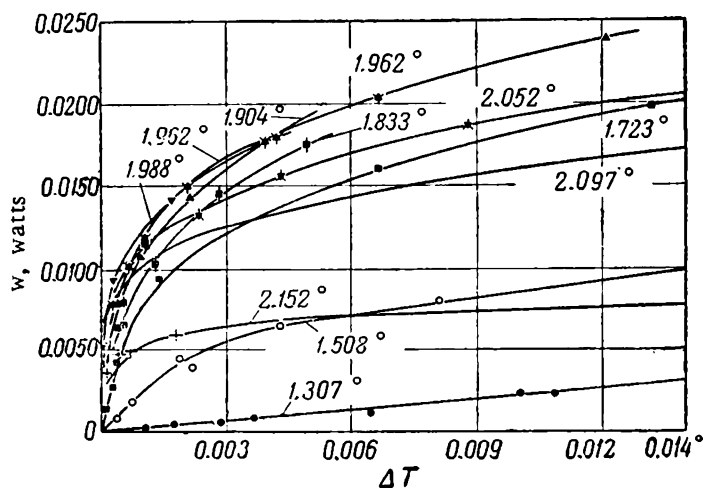
Fig. 41. Dependence of heat flow on temperature difference. Slit 10.5 μ.

In Figure 42 is shown the temperature dependence of the density of heat flow for slits of various widths at a constant value of grad T; it is apparent from this figure that the position of the maximum depends on the width of the slit: the narrower the slit the closer the maximum lies to the λ-point. For comparison, in this same figure are shown curves obtained with wide cylindrical capillaries.

A reduction in the temperature gradient also has an effect on the position of the maximum, moving it toward the λ-point. Both of these results are in disagreement with the data published earlier by Keesom, Saris and Meyer [23] which indicated that the position of the maximum is independent of the width of the capillary and the temperature gradient.

Representing the temperature dependence of the "thermal conductivity" analytically, in the relatively low temperature region Keesom and Duyckaerts obtained rather good agreement with experiment with grad T = 0.006 cm using empirically chosen values for the exponential factor for T (cf. Table 11).

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As is apparent from Table 11 the width of the slit has a particularly strong effect on the thermal conductivity when the gap lies between 19 and 9.3 μ. For capillaries of the order of 10 μ Allen and Reekie [21] found that the thermal conductivity was proportional to $T^{9.4}$.

TABLE 11

Thermal Conductivity as the Function of the Width of the Slit d

Width of the slit d, μ ...	0.75	1.15	9.3	19	100—700
Thermal conductivity. ..	T^{11}	$T^{11.6}$	$T^{12.8}$	$T^{6.63}$	$T^{5.4}$

The constant thermal conductivity with varying slit width found in the original work by Keesom and his colleagues is not corroborated in the experiments described here, as is apparent from Figure 42.

No less characteristic is the relation between heat transport and the fountain effect. Both Keesom and Duyckaerts and Mellink emphasize the direct proportionality between heat flow and fountain effect observed by them at all values of the applied power. This situation is illustrated by Figure 43 and Table 12 which are taken from the paper by Mellink.

It should be noted that aside from the proportionality between the difference in levels and heat flow (observed at a given temperature) there is a more complete parallel between the thermo-mechanical effect and the heat transport, in which the proportionality factor is a function of temperature.
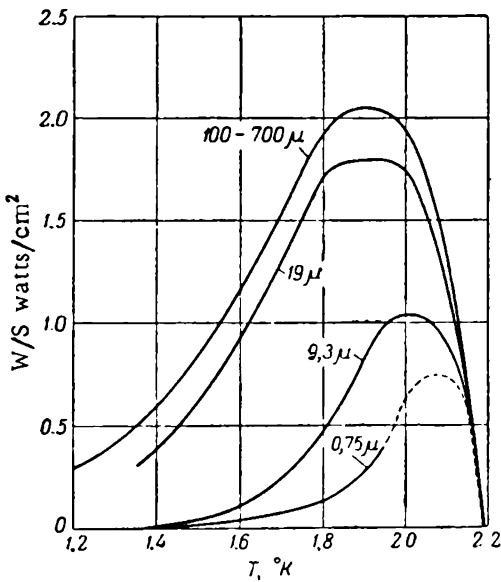

Fig. 42. Temperature dependence for heat transport in slits of different widths with grad $T = 6 \cdot 10^{-3}$ deg/cm. S is the cross-sectional area of the slit.

The experiments of Keesom and Duyckaerts on heat transport were carried out under conditions in which the thermo-mechanical effect should be marked (the absorption of heat by the entropyless helium flowing through the slit) and this was not taken into account

TABLE 12

Dependence of Heat Flow and Thermo-Mechanical Effect on Temperature Difference
(after Mellink; slit width 10.5 μ; slit length 2.48 mm)

T, °K mean	heat flow W, 10^{-4} watts	ΔT, 10^{-3} deg	ΔT, 10^{-3} (taken from uncorrected curves	level difference h, cm (exp.)	level difference h, cm (theoret.)
1.304	1.52	2 2	—	—	—
1.308	4 56	1.8	2.0	1.50	1.64
1.308	12.16	6.5	5.3	3.60	4.35
1.310	23.74	10.9	10.5	8.41	8.61
1.316	36.54	17.9	—	12.00	14.68
1.325	55.75	36.4	—	17.49	—
1.306	2.25	1.1	—	—	—
1.307	6.17	2.9	2 7	2.20	2.21
1.308	8.80	3 7	—	3.06	3.03
1.312	23.81	10.1	10.5	8.08	8.61
1.723	2.04	0	—	—	—
1.725	8.82	0	0.1	0.28	0.42
1.725	14.31	0.1_3	—	0.47	0.54
1.724	27.33	0.3	—	0.88	1.25
1.724	43.46	0.4	0.45	1.37	1.77
1.724	63.87	0.4	0.8	2.13	3.0
1.727	93.80	1.4	1.8	3.32	7.4
1.726	125.33	4.9	3.8	4 61	—
1.731	159.9	6.7	—	5.79	—
1.734	196.4	13.2	—	7.19	—
1.904	4.95	0.1	0.0	0.09	0.0
1.906	34.48	0.4	0.05	0.59	0.37
1.905	54.24	0.5	0.15	0.94	1.10
1.906	78.94	0.4	—	1.52	2.94
1.907	107.9	1.0	—	2.14	7.35
1.907	143.7	2.2	—	2.86	—
1.909	177.2	4 0	—	3 50	—
1.988	15.36	0.0	0.0	0.23	0.0
1.989	53.57	0.1	0.05	0.72	0.47
1.989	93.48	0.3_5	—	1.44	3.26
1.990	142.9	1.7_5	—	2.13	—
1.991	177.2	4.0	—	2.74	—

TABLE 12 (continued)

T, °K mean	heat flow W, 10^{-4} watts	ΔT, 10^{-3} deg	10^{-3} ΔT, (taken from uncorrected curves)	level difference h, cm (exp.)	level difference h, cm (theoret.)
2.097	17.32	0.0	0.0	0.19	0.0
2.097	42.05	0.0	0.0	0.52	0.0
2 093	80.96	0.0	0.3	0.93	3.7
2.099	119.1	2.3	—	1.22	—
2.108	177.1	20.8	—	—	—
1.508	2.03	0.42	0.1	0.20	0.19
1.508	8.03	0.49	0.32	0.72	0.62
1.509	18.10	0.77	—	—	—
1.509	39.34	2.2	—	—	—
1.514	79.38	8.2	7.8	7.31	15.1
1.520	117.8	21.1	—	10.21	—
1.514	63.76	4.4	—	5.67	8.40
1.514	45.11	1.9	2.3	3.26	4.4
1.832	12,49	0.13	0.05	0.28	0.30
1 833	20,32	0.0	0.1	0 64	0.60
1.833	65,85	0.6	0.42	1.56	2.50
1.835	145,5	2.9	—	3.69	—
1.836	175,4	5.0	—	4.58	—
1.835	103,0	1 4	—	2.66	8.15
1.834	79,8	0.6	0.7	1.93	4.2
1 961	46,92	0.0	0.05	0.67	0.44
1.960	60,73	0.0	0 1	0.92	0.87
1.962	79,48	0.3	—	1.31	2.44
1.962	102,6	0.7	—	1.76	5.92
1.963	120,0	1.1	—	2.03	—
1.963	179,3	4.2	—	3.25	—
1.965	203,6	6.7	—	3.81	—
1.967	233,9	12 1	—	—	—
1.964	150,8	2.1	—	2.56	—
2.052	51,9	0.4	0.2	0.78	2.20
2.052	83,0	0.6	—	1.40	6 9
2.053	114,6	1.1	1.4	1.76	—
2.054	155,7	4.3$_5$	—	2.37	—
2.056	185,9	8.9	—	3.77	—
2.054	132,3	2.3	—	2.04	—
2.068	209,1	20.5	—	—	—
2.152	49,01	0.71	—	0.78	—
2.159	76,57	15 0	—	—	—
2.153	60,15	1.85	—	1.02	—
2.152	47,24	0.44	0.8	0.73	—
2 152	35,14	0.18	—	0.50	—

by these authors. This oversight is most important for the narrow slits. Hence, it should be kept in mind that some of the numerical values obtained for the heat flow through the slit by these authors are in need of appropriate corrections.

NARROW SLITS. The system described in the preceding subsection was used by Meyer and Mellink [18] in making a careful examination of the thermo-mechanical effect in narrow slits. In essence these experiments consisted of the measurement of the difference in levels on both sides of the slit produced as a result of the flow of helium into the inner chamber, in which power was applied to the heater.

The slit length in these experiments was 1 mm. On different days the slit width was 0.15, 0.2, 0.3, 0.5 and 1 μ. The accuracy in the temperature measurement was 10^{-4} deg, the accuracy in the determination of the helium levels was 0.05 mm, and the accuracy in the determination of the volume

Fig. 43. Dependence of the fountain effect on heat flow. Slit 10.5 μ.

of the helium flowing in the capillary was 10^{-3} cm³. The greatest uncertainty in the results of the measurement was introduced by the slit width, which could not be ascertained with an accuracy better than 20%.

The characteristic dependence of the magnitude of the thermo-mechanical effect on the difference in temperatures at the two sides of the slit is shown in Figure 44; it is apparent from this figure that, as in the Kapitza experiments, the effect is found at zero load — a situation which reflects the imperfect thermal isolation of the inner chamber.

If the slit is narrow, the stationary state is established extremely slowly even though, in general, the fountain effect follows an increase in the difference in the temperatures between the two chambers. It is only with extremely narrow slits, at temperatures close to the λ-point, that the fountain effect ceases to follow the temperature
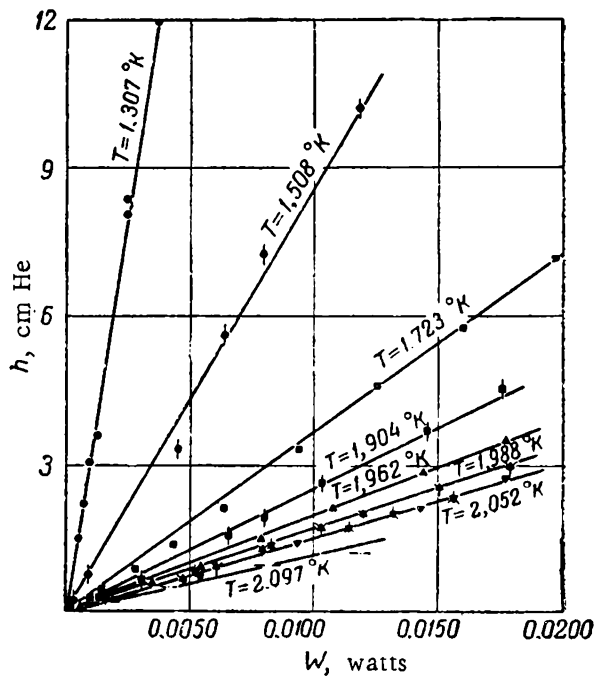
difference and noticeable relaxation effects are found. In this case the influx continues after the heater is switched off while the difference in temperature is gradually decreased.
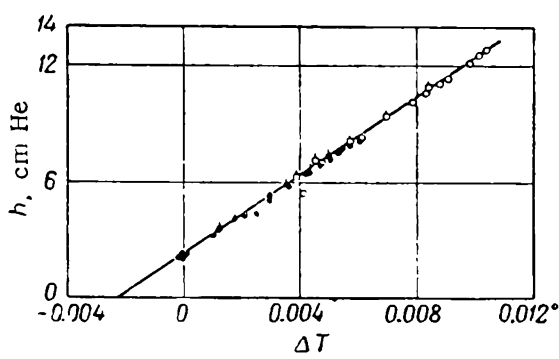

Fig. 44. Thermo-mechanical effect in narrow slits as a function of temperature difference. $T = 1.316°$K. Slit $0.5\,\mu$.

The authors have attempted to establish some value for the critical slit width. As has been explained, for a fixed average temperature and a given temperature difference the fountain effect is independent of slit width over a wide range. However, as the slit is gradually widened, a certain critical width is reached at the ends; at this width the effect falls off markedly. The higher the temperature the more rapidly the critical width is reached. A slit of "subcritical" dimensions is appropriately called "narrow" to distinguish it from "supercritical" or "wide" slits. It is found that the same slit can be narrow under certain conditions and wide for others. Table 13 shows critical slit dimensions at various temperatures.

TABLE 13

Critical Slit Width d at Different Temperatures

T, °K . . .	1.3	1.6	1.9	2.1	2.16	2 17
d, μ	10	5	2	1	0 3	0.1_5

In narrow slits, in which the fountain effect is most pronounced, a strict proportionality is observed between the pressure difference and the temperature difference. The temperature dependence of the thermal-mechanical effect is shown in Figure 45 from which it is obvious that this quantity is a monotonic function of temperature. The curve was taken for $\Delta T = 0.001°$. The various symbols refer to slits of various widths. In wider slits, at some temperature, which is lower the wider the slit, the effect differs from that observed in

narrow slits: after reaching a maximum it falls off rapidly, vanishing at the λ-point.

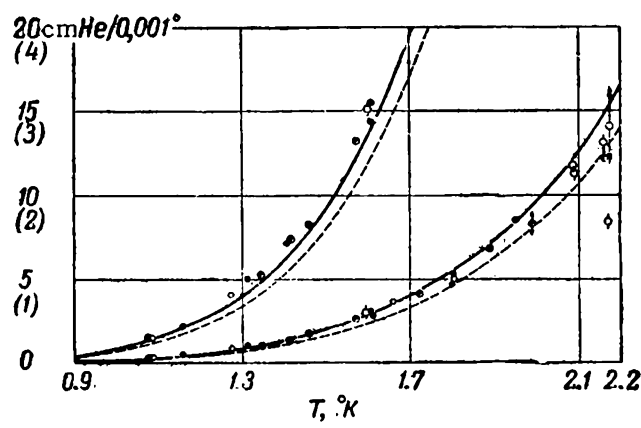It will be shown below that the critical slit width is intimately related to the critical velocity.

Fig. 45. Thermo-mechanical effect in narrow slits as a function of temperature. $T = 0.001°$. The different symbols refer to different experimental days and to different slits.

In addition to the experimental curve, in Figure 45 is shown also a theoretical curve (solid), computed with the London formula, which is similar to equation (9.9):

$$\frac{dp}{dT} = \rho S. \qquad (9.14)$$

The entropy values S in equation (9.14) were computed from the specific heat data of Keesom and Westmays [67] [sic] and Keesom and Keesom [24]. The accuracy in the entropy determination is 3%. At the lower temperatures the discrepancy between the experimental and theoretical values reaches 10%. The same discrepancy is found between the experimental results of these authors and the experimental results obtained by Kapitza who investigated the reverse effect (mechano-caloric effect) − the production of a temperature difference due to a pressure difference.

For comparison the authors show the dependence of the thermo-mechanical effect on temperature (dashed curve) computed using equation (9.14) in which the entropy is replaced by W/T (W is the enthalpy of the system).

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The authors note the incomplete reversibility of the flow process even in narrow slits. Without excluding of small linear velocities, the heat generated by the helium in flowing out from the internal chamber exceeds the value of ST (i.e., the heat absorbed in inflow) by the same 10%.

An irreversible effect was observed by these authors: this was a fluctuation in the temperature of the helium bath due to small variations of the level of the liquid in the capillary. This was observed even if $0.02 - 0.03$ cm^3 of the liquid would leak out of the chamber and then back; as a result of such a "single cycle" the temperature would increase by several ten thousandths of a degree in spite of the fact that it contained approximately 6 cm^3 of liquid helium. As has been indicated by Landau, this apparent irreversibility is due to the fact that the flow takes place from a container in which the helium II occupies a finite volume and has a finite free surface.

Within the limits of the discrepancies which have been pointed out, in the narrow-slit case the work of Meyer and Mellink is in good general agreement with the results obtained by Kapitza.

The numerical data for the thermo-mechanical effect taken from the work of these authors is shown in Table 14.

The dependence of heat flow on slit width, temperature difference, absolute temperature and other factors was investigated by Meyer and Mellink in narrow slits of the same width — from 0.15 to 1 μ. The length of the slit was 1 mm. Certain of the experiments were carried out under stationary conditions in which the difference in the levels of the liquid was not varied beforehand. However, because establishment of a stationary state requires an extremely long time, many of the measurements were carried out under nonstationary conditions in which the helium was first sucked in to the upper end of the capillary by the thermal load. A method of measuring the heat flow was also employed in which the inner chamber was gradually cooled by switching off the thermal load. Thus, in the present work all possible measures were taken to exclude errors. The results of these experiments are shown in Table 15.

In all experiments with fine slits the authors found a linear relation between the density of heat flow and the temperature difference

at all temperatures. The heat flow through the slit (thermal conductivity) was found to parallel the behavior of the fountain effect. The single exception was the dependence on slit width: whereas the fountain effect increased as the slit was reduced to critical dimensions the heat transport diminished.

TABLE 14

Magnitude of the Thermo-Mechanical Effect h, as a Function of Slit Width at Various Temperatures ($\Delta T = 0.001°$)

slit width d, μ	T, °K	thermo-mechanical effect h, cm He/0.001° (measured)	thermo-mechanical effect h, cm He/0.001° (calculated)
3.0	1.460	1.65	1.59
1.5	1.460	1.65	1.59
1.0	1.229	0.667	0.565
1.0	1.411	1.42	1.29
1.0	1.611	3.0	2.85
1.0	1.802	5.1 ± 0.2	5.4
1.0	2.160	12.2	14.7
0.5	1.086	0.304	0.272
0.5	1.160	0.44	0.40
0.5	1.274	0 819	0.702
0.5	1.316	1.00	0.85
0.5	1.659	3.65	3.35
0.5	1.816	5.06	5.64
0.5	1.966	7.89	8.75
0.5	2.116	10.8	13.0
0.3	1.226	0.692	0.557
0.3	1.600	3.03	2.76
0.3	1.804	5.16	5.42
0.3	2.042	9.0	10.7
0.3	2.089	11.6	12.1
0.3	2.160	12.6 ± 0 4	14.7
0.3	2.173	8.33	15.2
0.2	1.083	0.298	0.267
0.2	1.347	1.05	0.98
0.2	1.416	1.45	1.32
0.2	1.574	2.65	2.50
0.2	1.725	4.13	4.17
0.2	1 891	6.88	7.08
0.2	1.990	8.3 ± 0.4	9.33
0.2	2.152	14.9 ± 1.0	14.3
0.15	2.175	14 ± 3	15.3

It should be especially emphasized that in very narrow slits the heat flow increases as the temperature is increased to the λ-point, without showing any signs of a maximum at any temperature.

According to the authors the dependence of flow density on temperature is a function of slit width. For example, below 1.8° K the heat flow is proportional to T^4 for a slit of 0.15 μ, T^5 for a slit

of 0.3 μ, T^6 for a slit of 0.5 μ and T^9 for a slit of 1 μ. The authors interpret this finding by saying that the heat flow is not proportional to the capillary cross section. The absolute value of the thermal conductivity in narrow capillaries does not reach the high values observed in wide capillaries; however it is still many times greater than the thermal conductivity of helium I, exceeding the latter by a factor of $10^5 - 10^6$.

TABLE 15

Heat Transport (heat flow W) in Helium II Through Narrow Slits as a Function of Slit Width d at Various Temperatures; $\Delta T = 0.001°$

slit width d, μ	T, °K·	heat flow W, 10^{-4} watts (stationary case)	heat flow W, 10^{-4} watts (nonstationary case)
1	1.411	0.220	0.240
	1.611	0.46	
	1.802	2.6	
	1.948	4 5	3.9
	2.159	6.5	
0.5	1.086	0,013	
	1.089	0,018	
	1.274	0 029	
	1.315	0.037	
	1.659	0.260	
0 3	1.226	0.016	
	1.358	0.030	
	1.558	0.085	
	1.605	—	0.057
	1.652	0 120	
	1.842	—	0.121
	2.090	—	0.257
	2.161	—	0.290
0.2	1.070	0.003	0.009
	1.088	0.008	
	1.582	—	0.024
	1.733	—	0.035
	1.896	—	0.045
0.15	1.932	—	0.037
	1.070	0.006_8	0.006_8
	1.502	—	0.029
	1.901	—	0.058
	2.100	—	0.072

According to Meyer and Mellink there are two types of thermal conductivity in helium II: one is characteristic of wide slits and capillaries while the other comes into play in narrow capillaries and slits. In wide capillaries (diameters ranging from 0.01 to 0.1 mm)

the heat flow is proportional to the cross section and does not depend on length as long as the quantity $l/\delta T$ remains constant. The heat flow is proportional to the cube root of the temperature difference which exists between the ends of the capillaries. Up to temperatures of $1.6°$ K it increases as T^5. At $T = 1.9°$ K there is a maximum in the thermal conductivity at which its value is a factor of 10^8 greater than the thermal conductivity of helium I. As the temperature is increased beyond this point the thermal conductivity falls off rapidly.

In narrow capillaries the thermal conductivity is such as to cause a departure from the strict linear relation between heat flow and temperature difference. The dependence of heat flow on temperature for slits of varying widths has been described in the present section.

In the intermediate region (slits of width $1 - 30\,\mu$) Meyer and Mellink propose that both forms of the thermal conductivity are effective and that each is observed in a smaller or greater degree depending on the absolute temperature, the width of the slit and the temperature gradient.

DISCUSSION OF THE RESULTS.   All the seemingly complicated features characteristic of heat transport and the thermo-mechanical effect are easily interpreted by means of the hydrodynamic ideas embodied in the Landau theory. The application of the hydrodynamic equations to heat transport is based on the assumption that the superfluid component of helium II, which carries zero entropy, moves in the direction of the heat source. On the other hand, the normal part of the liquid is presumed to move away from a heat emitting surface, carrying the generated heat with it.

The mean velocity of the normal component can be easily computed from equation (9.11) given in Section 5

$$\bar{v}_n = \frac{\varphi}{\rho Q}.$$

If the heat transport takes place in a sufficiently narrow slit, as is the case for example in the experiments being described here, two conditions must be applied in equation (9.11): one of these is the equation for the thermo-mechanical effect (9.14), already known

to us, which relates the difference in temperatures produced at both sides of the slit to the pressure difference:

$$\frac{dp}{dx} = \rho S \frac{dT}{dx}.$$

The second condition arises from the fact that the motion of the normal component through a narrow slit, which at relatively small velocities can be assumed laminar flow, satisfies the usual Poiseuille equation. In the particular case of a plane slit the pressure gradient is given by the equation

$$\frac{dp}{dx} = \frac{12\eta\bar{v}}{a^2} = \frac{12\eta}{a^2} \frac{\varphi}{\rho Q}, \tag{9.15}$$

where a denotes the width of the slit and η is the viscosity of the normal component.

Combining all three equations we find that the heat flow is a linear function of the temperature gradient:

$$W = \frac{2\pi a^3 R \rho^2 Q^2}{12\eta_n T} \frac{dT}{dx}. \tag{9.16}$$

Here W is the total heat flow, which is equal to $W = 2\pi a R \varphi$, where $2\pi R$ is the perimeter of the circular slit used by Keesom and others.

It is apparent from these formulas that the empirical results obtained to this point for wide slits i.e., the nonlinear relation between heat flow and temperature difference, the maximum in the heat transport, and the proportionality of the flux to the cross sectional area of the heat conductor are not accounted for by the theory. However, an exact quantitative comparison of the experimental data on heat transport with the predictions of the theory is difficult because of the absence of information on the true temperature dependence of the viscosity of the normal component.

At the present time we have at our disposal all the data necessary for a theoretical analysis of the experimental results described in the preceding subsections. However, in view of the fact that the heat transport measurements of Keesom and Duyckaerts [22] are susceptible, as has been already mentioned, to serious errors in method, the results of this work are not very useful for theoretical analysis.

We consider first the Mellink experiments [13] with slits of relatively large dimensions — 10.5 and 5 μ. An examination of the dependence of heat flow on temperature difference in the vicinity of the origin leads to the conclusion that this dependence is strictly linear. It may be noted that at low heat loads no maximum is observed in the thermal conductivity. However, when some critical temperature difference is reached the transported heat increases much more slowly and in this region the temperature dependence of the heat flow exhibits a clearly defined maximum, such as observed in the earlier work. A computation of the velocity of the relative motion between the normal and superfluid components indicates that the linear relation between heat flow and temperature difference no longer holds when the velocity reaches values of 15 — 20 cm/sec; these values are in agreement with the critical velocity values observed by a number of investigators in the flow of helium II along films, in capillaries, and in slits.

Thus, even a superficial analysis of the Mellink data indicates that the heat transport process takes place in accordance with the theoretical predictions and that the observed discrepancies from the theory must be attributed to the fact that the critical velocity is exceeded.

The agreement between theory and experiment is actually more complete: substituting the Mellink data in equation (9.16) with the experimental values obtained by Kapitza [16] and Andronikashvili [12] for the heat content and the viscosity we obtain a result close to that obtained by Mellink (within the limits of the experimental errors). The discrepancies observed for the 10.5 μ slit, which are as high as 100%, may be explained by the inaccuracy in the determination of the slit dimensions. As has been indicated this error may be as large as 20%. Since the width of the slit is raised to the third power in equation (9.16) the indicated error can lead to a discrepancy of 70% even without other errors.

In Table 16 are shown some of the experimental data taken from Mellink and the values of the heat flow computed from equation (9.16). For comparison purposes the low load region has been chosen since in this region the laws of superfluid motion are known to apply.

Thus, a comparison of the experimental and theoretical results leads to the important conclusion that for the present type of slit

TABLE 16

Comparison of the Experimental and Theoretical
Values of Heat Transport Computed from Equation
(9.16)

(Mellink data)

slit		T, °K	ΔT, 10^{-3} deg	W, 10^{-4} watts experimental	W, 10^{-4} watts theoretical
width, μ	length mm				
10.5	2.48	1.325	36.4	55.75	130
		1.724	0.3	27.33	62.5
		1 724	0.45	43.5	93.2
		1.508	0.32	8.03	11.2
		1 832	0.05	12.49	20.7
5	1 00	1.328	13.1	16.38	13.9
		1.714	0.6	25.8	30.9
		2.032	0.01	24.52	28 9

the heat transport process in helium ll is described quantitatively by the Landau theory so long as the heat transfer is "subcritical." It should be noted that the presence of a maximum in the "heat flow — absolute temperature" curve indicates that the critical velocity is exceeded. As far as the position of this maximum on the temperature scale is concerned we can only say that it depends on a number of factors and is apparently not a characteristic feature of the heat transport process in helium ll.

We now turn to an analysis of the results obtained by Mellink in observations of the thermo-mechanical effect. From the formulas which have been given it is obvious that the heat flow through a slit is determined in equal degree by the difference in temperature and the difference in pressure; this situation is completely reasonable in view of the fact that the hydrostatic pressure and the temperature pressure are completely equivalent. Hence it is natural to expect that under conditions for which the proportionality between the heat flow and temperature difference is disturbed there should also be a disruption of the linear relation between the heat flow and the magnitude of the thermo-mechanical effect.

Both Keesom and Duyckaerts, and Mellink emphasize that the direct proportionality between the heat flow and the difference in liquid level on both sides of the slit is observed at all loads. lt should

be noted that this relation, which does not take into account the additional pressure due to the difference in vapor pressures (because of the different temperatures on both sides of the slit) is not physically meaningful and is entirely coincidental.

Thus, Table 13 and the curve in Figure 43 require appropriate corrections. The correct values for the thermo-mechanical effect are given in Table 17. A characteristic dependence of heat flow on pressure difference (slit of 10.5 μ, T = 1.725° K) is shown in Figure 46; it is apparent that on transition through some critical load value the total pressure difference $H = h + h_1$ exhibits a marked change (here h is the difference in levels and h_1 is the difference in vapor pressures expressed in centimeters of helium). The curves which have been plotted, which are similar to those in Figure 152 of *Helium* by W.H. Keesom, can be used to obtain information as to the critical velocities. Actually, taking into account the fact that the mean velocity of the normal component is determined from equation (9.11) and that in a plane slit the maximum velocity $v_m^{(n)}$ is one and a half times greater than the mean velocity, we find

$$v_m^{(n)} = \frac{1.5 \varphi_c}{\rho Q},$$

where φ_c is the critical value of the heat flow density. On the other hand, the velocity of the superfluid component, which is uniform over the entire cross section of the slit, is determined from the value of the mean velocity of the normal component by the relation

$$v_s = \bar{v}_n \frac{\rho_n}{\rho_s}.$$

Substituting the appropriate numerical values for the quantities which appear in the expression for $v_m^{(n)}$ and v_s and combining these two velocities we find the critical velocity:

$$v_c = \frac{\varphi_c}{\rho Q} \left(1.5 + \frac{\rho_n}{\rho_s} \right).$$

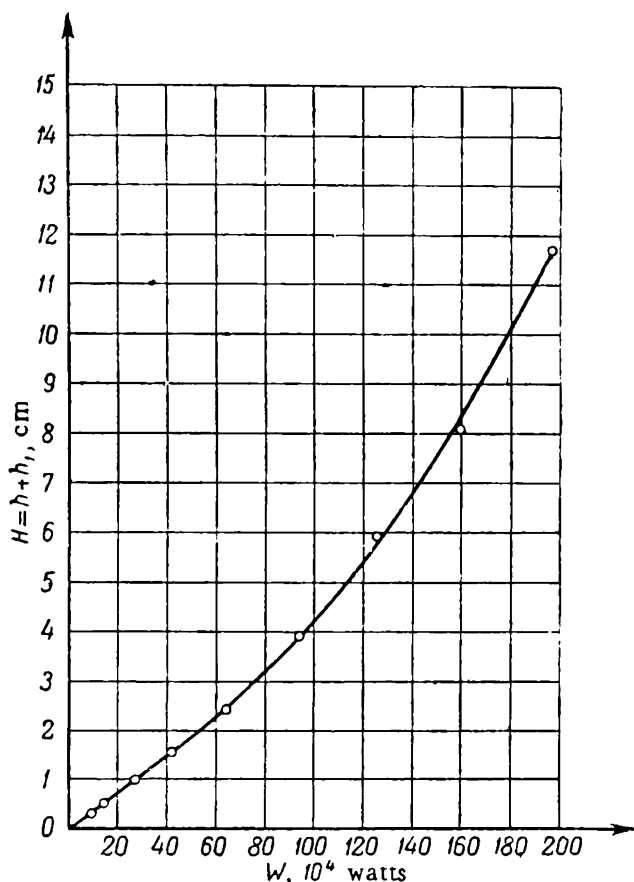which, for the case shown in Figure 46, yields a value of 10 − 11 cm/sec (cf. Table 10).

Fig. 46. Dependence of the fountain effect on
heat flow. (Corrected Mellink data.).

Thus, we see that the maximum thermo-mechanical effect which can be obtained without disturbing the linear relation between the temperature difference and the pressure difference is determined (for a given slit width and absolute temperature) by the critical velocity.

In this connection the motion of a critical slit width, first introduced by Meyer and Mellink [18], becomes clear: a critical slit is a slit for which the transition through the critical velocity occurs at the given values of temperature and the temperature difference.

The existence of a maximum in the dependence of the thermo-mechanical effect on absolute temperature is also associated with the transition through the critical velocity. As in the temperature dependence of the heat transport, the position of the maximum on the temperature scale is to some extent coincidental and is not of

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direct physical significance in connection with the processes which take place in helium II.

TABLE 18

Thermo-Mechanical Effect and Viscosity of the Normal Component as a Function of Heat Flow
(calculated from the experimental data obtained by Mellink)

slit width μ	length mm	T, °K	W, 10^{-4} watts	ΔT, 10^{-3} deg	h, см	$H=h+h_1$, см	η, 10^{-5}, poise	remarks
10.5	2.48	1.325	55.75	36.4	17.49	20.25	2.83	
		1 725	8.82	0.1	0.28	0.315	1 71	$\bar\eta=1.71\cdot$ $\cdot10^{-5}$ poise
		1.725	14.31	0.13	0.47	0.516	1.73	
		1.724	27.33	0.3	0.88	0.985	1.73	
		1.724	43.5	0.45	1.37	1.53	1.69	
		1.724	63.87	0.8	2.13	2.41	1.81	
		1.727	93.80	1.8	3.32	3.95	2.02	beyond critical region
		1.726	125.3	3.8	4.61	5.94	2.26	
		1.731	159.9	6.7	5.79	8.13	2.44	
		1.734	196.4	13.2	7.19	11.74	2.87	
		1.508	8.03	0.32	0.72	0 77	1.92	beyond critical region
		1.514	63.76	4.4	5.67	6.33	1 99	
		1.514	79.38	7.8	7.31	8.48	2.14	
		1.520	117.8	21.1	10.21	13.37	2.27	
		1.832	12.49	0 05	0.23	0.305	1.73	
5	1.00	1.328	16.38	13.1	12.4	13.14	1.49	
		1.714	25.8	0.6	2.70	2 91	1.18	beyond critical region
		1 716	72.6	4.2	7.95	9.41	1.36	
		1.725	124.3	21.5	14.13	21.6	1.83	
		2.032	24.52	0.01	1.16	1.17	1.43	
		1.220	0.412	1.2	0.76	0.83	2 20	$\bar\eta=2.18$ $\cdot10^{-5}$ poise
		1.222	1.87	5.1	3.00	3.30	1.92	
		1.213	3.17	12.8	6.18	6.92	2.38	
		1 211	2.32	8.8	4.51	5.02	2.36	
		1.218	6.65	22.5	11.1	12.41	2.04	

Having established the validity of equation (9.16) for heat transport, we use equation (9.15) for determining the temperature dependence of the viscosity of the normal component and to determine the manner in which this quantity changes beyond the critical region. In Figure 47 is shown a typical dependence of the viscosity as a function of heat flow obtained from an analysis of the experimental data reported by Mellink. In the region in which the linear relation

between the heat flow and thermo-mechanical effect obtains the viscosity of the normal component, as is to be expected, remains constant. When the critical load is reached the viscosity increases markedly; this increase is to be associated with the appearance of
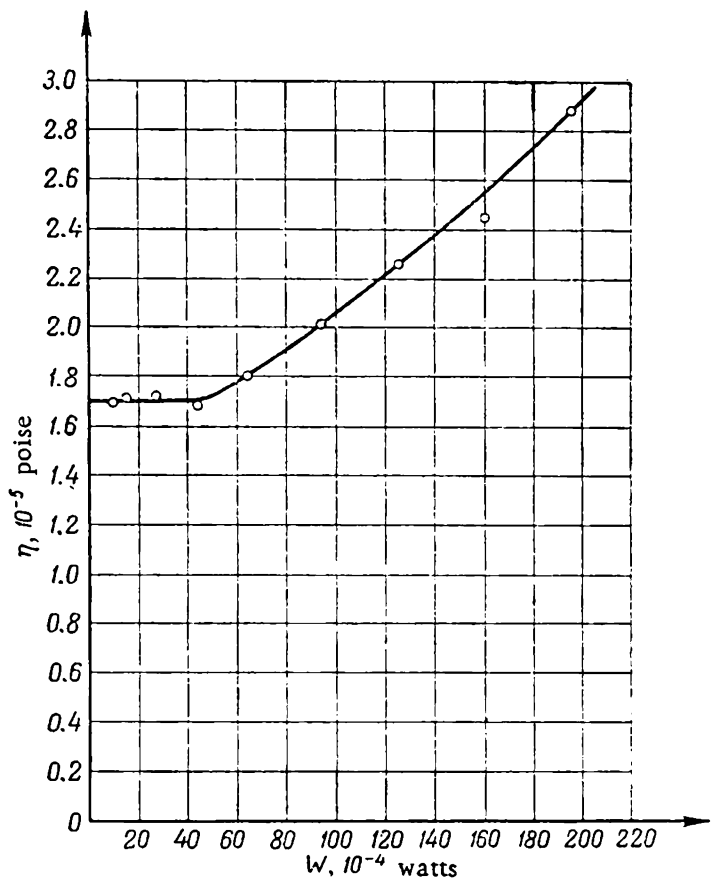

Fig. 47. Viscosity as a function of heat flow. $T = 1.725°\,\mathrm{K}$. Slit $10.5\,\mu$. (Computed from the experimental data obtained by Mellink.)

irreversible losses. The origin of the discrepancies between the numerical values of the viscosity measured by Ankronikashvili [12] and those computed using the Mellink data have already been analyzed in Section 3.

As has been noted in Section 3, the deviation of the viscosity values obtained with a slit $5\,\mu$ wide, observed at temperatures below $1.5°\,\mathrm{K}$, is apparently due to the long range of the quasi-particles which take part in the heat transport. The Knudsen effect and slipping

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which is associated with it, have a pronounced effect in slits of width ranging from 1 to 0.15 μ. Figure 48 shows the temperature dependence of the apparent viscosity in slits of width 1, 0.5 and 0.3 μ, computed from the data of Meyer and Mellink.
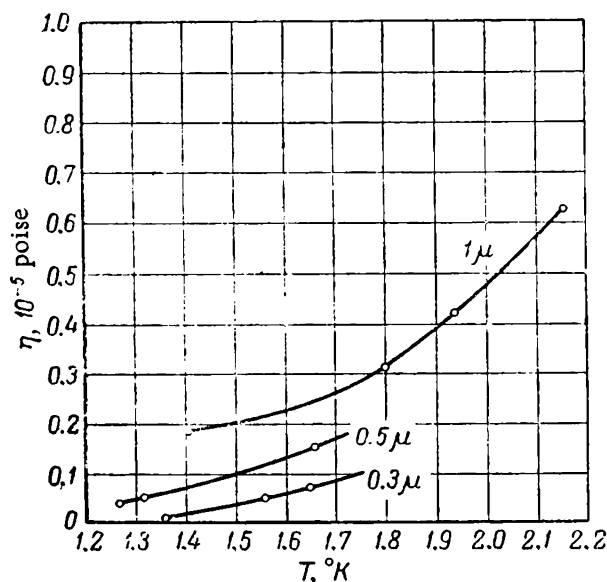
Fig. 48. Apparent value of viscosity in narrow slits. The slit width is indicated on the curves. (Computed from the experimental data obtained by Meyer and Mellink.)

In heat transport in narrow slits the experiment gives values which are somewhat greater than are expected on the basis of theoretical considerations. Table 18 gives an idea of the discrepancy between the experimental and theoretical values observed in different slits at various temperatures. It is apparent that the difference between the values given in the second and third columns of the table is approximately proportional (particularly for slits of 0.3 and 0.5 μ) to the first power of the slit width. Thus, the true heat transport in narrow slits is made up of two terms: a term which is cubic in slit width and a term which is linear. The origin of the second of these terms has still not received a rigorous quantitative explanation.

In the light of our analysis of the experimental data obtained in the work of Keesom and Duyckaerts, Meyer and Mellink, and Mellink, the attempts made by these investigators to suggest a particular form of the temperature dependence of the heat transport for each of the slit dimensions is completely unnecessary. The same applies to the proposal that there are two different thermal conductivities in helium II. In the region in which the superfluid motion obtains, there is only one type of heat transport. However, two factors may tend to complicate this phenomenon: on the one hand we have the transition through the critical velocity, in which case irreversible losses arise; on the other we have slipping, which is associated with the behavior of the gas of thermal excitations in the Knudsen region.

TABLE 18

Comparison of the Experimental and Theoretical
Values of Heat Transport in Narrow Slits
(from the data of Meyer and Mellink)

T, °K	d, μ	W, 10^{-4} w experimental	W, 10^{-4} w theoretical	$\Delta W = (W_{\text{exp.}} - W_{\text{theor.}})$ 10^{-4} watts
1.4	1	0.199	0.023	0.176
	0.5	0.060	0.0023	0.057
	0.3	0.038	0.0006	0.037
1.5	1	0.417	0.066	0.351
	0.5	0.107	0.0082	0.099
	0.3	0.063	0.0018	0.061
1.6	1	0.832	0.169	0.663
	0.5	0.182	0.0212	0.161
	0.3	0.098	0.0047	0.093
1.7	1	1.51	0.366	1.14
	0.5	0.316	0.0456	0.270
	0.3	0.140	0.0098	0.130

Section 7. Heat Transport in Free Helium II

INTRODUCTION. In many papers concerned with helium II one
finds the assertion that the anomalously high heat transport is due
to the presence of wall films, along which an especially easy trans-
port of heat can take place. In this connection it is also assumed
that the free or "bulk" helium does not have this particular property
and that the heat transport in bulk helium is very much the same as
in helium I. Originally Kapitza [4] subscribed to this notion, on the
basis of a calculation of the "classical thermal conductivity" of free
helium. These statements as to the peculiar properties of wall films
in helium II are still encountered at the present time although there
is now little doubt that the use of the term "thermal conductivity"
in connection with helium II is not suitable since classical con-
siderations are not applicable.

OPTICAL OBSERVATIONS OF THE HEAT TRANSPORT PRO-
CESS. Although the optical properties of liquid helium are essen-
tially independent of temperature, under certain conditions one might
hope to observe ordinary convection currents if these actually exist

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in helium II. The sensitivity of a method developed for this purpose by Strelkov [25] was such that it would make it possible to observe the existence of currents in which the temperature differed from the temperature of the medium by 0.02°. Strelkov constructed a special optical dewar for this purpose: this was a single four-walled container with two pairs of flanges cemented to the inside and outside of plane-parallel slabs of optical glass. The dewar is shown in Figure 49. On one side of the dewar there is a point — source of light and on the other there is either a screen for visual observation or a movie camera.

Inside the dewar there are three electric heaters: a plane disk 6 mm in diameter, a constantan wire 0.04 mm in diameter and 40 mm long stretched in the free helium on a bracket, and a constantan wire of the same dimensions clamped between two quartz slabs 20 mm in diameter. The gap between the slabs is made as parallel as possible.

to pump
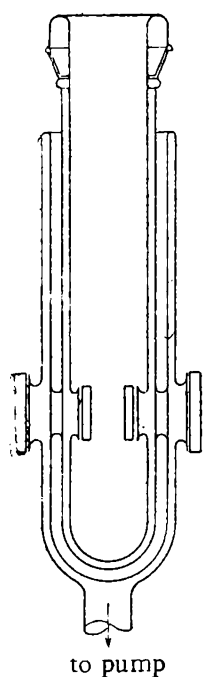
Fig. 49. Optical dewar used by Strelkov.

At small loads no traces of any optical inhomogeneities are found. At high loads a marked inhomogeneity is observed and this was due to the appearance of the gas phase around the heater. The heater is surrounded (in particular the thin wire) by a cylindrical, sharply-defined envelope. The envelope dimensions increase smoothly as the load is increased, and when its diameter is of the order of 0.5 mm exhibits signs of instability, characterized by the ejection of gas jets.

The apparent conductivity in helium II is defined by the expression

$$k = \frac{\varphi \ln \frac{l}{2r}}{2\pi (T - T_0)}, \qquad (9.17)$$

where k is the thermal conductivity, φ is the heat emitted per unit length, l and $2r$ are the length and diameter of the gas cloud, T is its temperature and T_0 is the temperature of the medium. The temperature at the surface of the gas envelope is computed from the pressure of the saturated vapors above the liquid, the hydrostatic

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pressure, and the capillary pressure. In view of the fact that the accuracy of these measurements is relatively poor, Strelkov quoted an apparent thermal conductivity between 10 and 30 watt/cm·deg; this is several orders of magnitude smaller than the value obtained by other authors for the capillary case.

Observing the formation of the gas envelope in heating the wire clamped between the slabs, Strelkov noted that the formation of the vapor phase always takes place from a somewhat superheated liquid. The magnitude of this superheating, unfortunately, was too small to be measured. A rough estimate was made in order to determine the order of magnitude of the thermal conductivity; in the present experimental conditions a figure of $2-5 \cdot 10^3$ watts/cm·deg was obtained.

The disagreement in the results obtained with different versions of the same method is interpreted by the author as evidence of the incorrectness of the proposal made by Keesom to explain the high value of the true thermal conductivity in helium II. Howsoever rough the estimates carried out by Strelkov, the difference in the thermal conductivity obtained for the freely suspended wire and the clamped wire are additional evidence of the peculiar convective nature of the heat transport which takes place in free helium ll.

TEMPERATURE DISTRIBUTION CLOSE TO A HEAT DISSEMINATING SURFACE. As has been shown in Section 1, heat transport in helium II which fills a capillary is so large that at low loads in most cases one cannot observe a temperature differential at the end of the capillary because the effect lies beyond the limits of the experimental accuracy. The experiments described in Section 1 and Section 5 of the present chapter, which establish the nature of heat transport in helium ll, however, give no indication of the temperature distribution near a heat disseminating surface.

Kapitza carried out an experiment devoted to this problem [4].

A spiral of phosphor bronze wire is wound on a glass frame and used as a resistance thermometer (Figure 50). When a beam of light illuminates the thermometer a noticeable increase in temperature is observed in spite of the fact that the entire system is freely cooled by helium II. An estimate of the applied power lead Kapitza to the conclusion that the heat transport in the surrounding helium

is not anomalously high. Experiments carried out at various temperatures for the helium bath but with the same power applied to the spiral showed no maximum in the thermal conductivity at 1.9° K as was observed in capillaries.
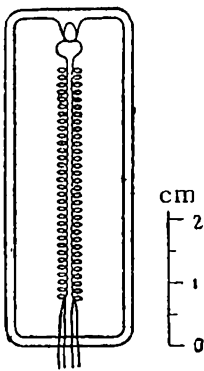

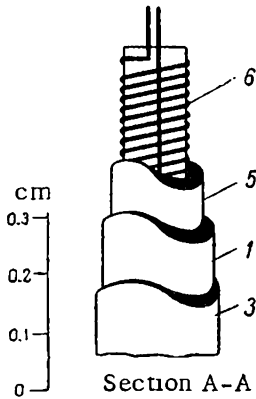Fig. 50. Device for studying heat transport in free helium II.

Section A-A

Fig. 51. Device for studying heat transport in free helium II. Different heater construction.

In another experiment [4] (cf. Figure 51) the heater 1 is a platinized glass tube 6.5 cm long and 1.85 mm in diameter. The current and voltage leads 4 are connected to a platinum layer 3. A bronze thermometer 6 is placed in the tube and surrounded by a superconducting magnetic shield 5 made of lead. The tube is filled with gaseous helium at atmospheric pressure, hermetically sealed, and suspended in the frame 2 by means of the current carrying conductors. As is apparent from Figure 52, the temperature at the surface of the heater is a linear function of the applied power; for a constant load the temperature difference ΔT is larger, the lower the temperature of the bath.

As in the preceding experiment, no maximum in the thermal conductivity is observed at 1.9° K.

In the author's opinion this experiment makes possible a reasonably accurate numerical estimate of the apparent thermal conductivity of free helium II. For example, at $T = 1.695°$ K, from classical considerations the value of k is 0.61 watts/cm·deg, that is, approximately 15 times smaller than that found in the Strelkov experiment [25]. However, this quantity is still 100 times greater than the

thermal conductivity of helium I. In this work it is suggested that if we do not invoke convection currents in free helium II, the thermal conductivity is not different from the thermal conductivity of helium I.
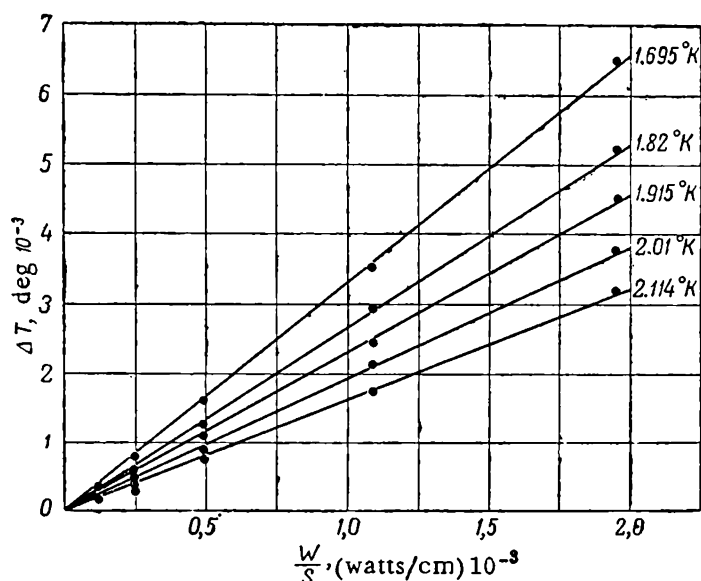

Fig. 52. Dependence of the difference in temperature between a freely suspended heater and helium as a function of applied power at different temperatures.

Further investigations of heat transport in free helium II were carried out with the system shown in Figure 53 [4]. The heat disseminating surface in this case is the surface of the parallelopiped 1 made from highly annealed pure copper. In the parallelopiped there are two internal cavities with openings in opposite directions. In one cavity there is a bifilar heater 3; the other cavity contains a bronze thermometer 2. The potential and current connections are made through the glass headers 4 and 5 sealed to the copper. The cavities were filled with gaseous helium at atmosphere pressure.

Two such parallelopipeds, placed as close together as possible, are suspended in the helium II by fine wires.

If the heater is turned on in one, it is possible to measure an increase in the temperature in the other although this increase is never more than $5 - 10\%$ of the temperature increase of the parallelopiped in which the heat is generated. On the basis of these

experiments, Kapitza concluded that the temperature differential is all in a thin layer of helium at distances of the order of a hundredth of a millimeter from the surface giving off the heat.
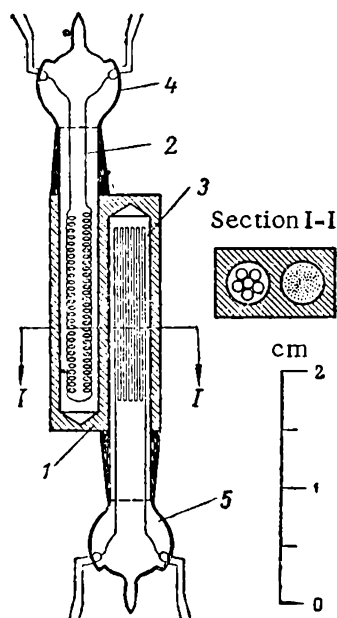

Fig. 53. Device for studying the temperature differential in helium as a function of the state of the surface.

Fig. 54. "Thermal conductivity" of a wall layer of helium as a function of temperature.

Since an exact determination of the thickness of this layer would be extremely difficult, Kapitza introduced the characteristic thermal conductivity, defined by the expression

$$t = \frac{W}{S\Delta T} \text{ watts/deg. cm}^2 \tag{9.18}$$

where W is the heat flow and S is the surface of the heater. Figure 54 shows the temperature dependence of the thermal conductivity of the layer. The different curves refer to surfaces which were treated in different ways.

DISCUSSION OF THE RESULTS. A systematic application of Landau's ideas leads to an interesting interpretation of the Strelkov

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experiments[1] in which measurements were made of heat removal from wires heated by electric current; one of the wires was freely cooled by helium II while the other was clamped between two plane supports [25]. With the application of a certain limiting load to a wire 0.04 mm in diameter, as has already been indicated a gaseous envelope is formed whose diameter reaches 0.5 mm. In the case of the freely cooled wire there is a sharp change in the size of the heat removal surface. On the other hand, in the wire which is clamped between glass slabs, there is no change in heat removal due to the envelope; this surface remains equal to twice the cross sectional area of the gap between the slabs. Since the velocity of the normal component, which carries the heat, is determined by the ratio of the load to the heat removal surface, all other conditions remaining equal, the value of the velocity of the normal component is characteristic of each of the cases being considered. A very rough recalculation of the Strelkov data for similar temperatures, leads to the following results: W_1/S_1 = 4 watts/cm^2 for a wire which is in free contact with the helium (W_1 is the heat flow, S_1 is the surface of the gas envelope), W_2/S_2 = 7.5 watts/cm^2 for a wire clamped between the slabs (W_2 is the heat flow and S_2 is the cross sectional area of the slit).

Thus we see that velocities of the normal component corresponding to approximately the same conditions are very much the same if account is taken of the large experimental errors and the rough approximations used in the estimates (the fact that the temperatures are not exactly the same, the uncertainty in the determination of the diameter of the gaseous envelope and so on). It might appear that experiments such as these might be used to obtain information as to the magnitude of the critical velocity. However, there is a great deal of evidence to indicate that the gaseous phase arises from considerable superheating of the liquid phase. This means that the limit of nonviscous flow is exceeded. Thus, under the conditions for which a gaseous envelope is produced it is impossible to draw any conclusions as to the critical velocities.

However, all the numerical estimates made here in connection with experiments on heat removal are interesting; they serve as a

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[1]The application of this interpretation to the Strelkov experimental results was first made by Peshkov [26].

convenient method of indicating the fallacy in the notion of apparent thermal conductivity and show how to avoid the contradiction between measurements of the thermal flow obtained under different conditions. (As has been indicated above, the apparent thermal conductivity varies by at least a factor of 100 in the various experiments performed by Strelkov.)

We now turn to the Kapitza experiment [4] devoted to a study of heat removal in free helium II. Of all the experiments described above, only one, that shown in Figure 51, contains all the data necessary for numerical calculations. As is apparent from an examination of Figure 52, which illustrates the results of this experiment, even at relatively weak specific loads (of the order of $2.5 \cdot 10^{-4}$ watts/cm^2) the temperature rise inside the body being heated is .001°. The relative velocity of the normal component and superfluid component corresponding to this specific heat is hardly more than a hundredth of a millimeter per second, i.e., it is well within the subcritical region. It is found that the relation between the observed temperature difference and the specific load is strictly linear.

The difference in temperatures of the heated body and the helium which fills the cryostat is found to be greater as the temperature of the cryostat is reduced. The reasonable assumption that the factor responsible for the temperature increase is the delay in the formation of thermal excitations, as indicated by Landau, leads to a relaxation time of the order of $10^{-1} - 10^{-2}$ seconds which is also too large. In this connection Landau has proposed that the origin of the retarded heat removal lies in the reflection of thermal waves at the surface of the solid body; this reflection takes place at the interface between the solid surface and the helium II which is in contact with it. This point of view tends to be supported by the fact that the heat removal is affected by the state of the surface, in particular by the nature of the material which comes into direct contact with the helium II (cf. Figure 54). These last remarks are of purely qualitative nature and are introduced here only to suggest one possible explanation for the effects which are observed.

We now turn to a consideration of the experiments which were discussed in Section 1.

We first direct our attention to the fact that the pressure exerted on the vane of the radiometer by the thermal flow is approximately

a quadratic function of the applied power. This situation appears to be general and applies both to the case of free helium as well as helium which flows from a capillary. In the absence of the necessary data it is impossible to make a comparison of the results obtained by Strelkov [27] with the predictions of the theory. However, the results of the Kapitza experiments [4] are in complete accord with the theoretical interpretation. As a matter of fact, if we recall the fact that the motion of the superfluid component satisfies the law for potential flow of an ideal liquid, we can ascribe the entire experimentally measured effect to the normal component. Neglecting friction at the walls of the capillary it may be assumed that the pressure in the jet is equal to the momentum
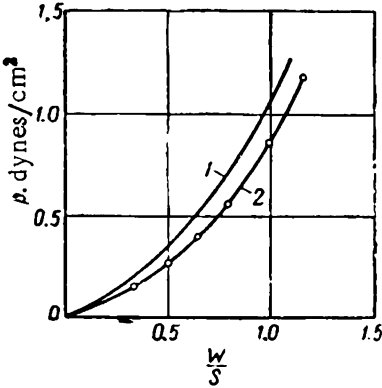
Fig. 55. Pressure of the jet sucked out of the capillary as a function of load. (Comparison between experimental data and theory.) $\frac{W}{S}$ is given in watts per square centimeter.
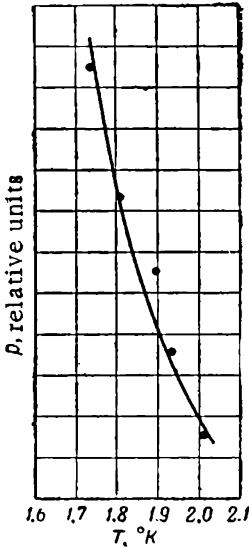
Fig. 56. Reactive force on the radiometer as a function of temperature. (Comparison between experimental data and theory).

flow density, that is ρv_n^2. Thus, the pressure on the vane of the torsional weights or the radiometer must actually be proportional to the square of the velocity of the normal component or, as follows from equation (9.11), the square of the specific load.

A comparison of the numerical data obtained in the Kapitza experiment with the theoretical data in the small load region where critical velocities should not have an effect, allows us to plot the curves in Figure 58 (compare this with Curve 5 in Figure 12). The small discrepancy, of the order of $30 - 35\%$, may be completely attributed to the experimental errors or errors made in estimating the tolerances.

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In order to demonstrate that the heat transfer process in free helium is essentially the same as that in the capillary it is important to compare the experimental and theoretical temperature dependence of one of the physical quantities associated with the heat removal mechanism. One such quantity is the pressure exerted on the vane of the radiometer.

In view of the fact that at a fixed load the velocity of the normal component depends only on the heat content of the helium II, the momentum flow density is given by an expression which is proportional to ρ_n/Q^2. Inasmuch as ρ_n and Q are both exponential functions of the temperature, the momentum flow density (or pressure) should increase as the temperature is reduced; this increase should be exponential and not linear as has been indicated by Strelkov. A detailed analysis of Figure 23 is basically in agreement with this conclusion (cf. Figure 56) although it is impossible to make a quantitative comparison of the experimental and theoretical results because of the absence of the required numerical data.

It should be noted that the flow of thermal quanta in free helium II is governed by all the relations which apply for the so-called "submerged jet." As has been shown by Landau [28] the spreading of this jet should increase as the density and viscosity of the liquid are increased.

Such an effect has actually been observed experimentally if one recalls how rapidly the spreading of the jet in the Kapitza measurements increased as the λ-point is approached (cf. Figures 14 and 15); close to this point the density and viscosity of the gas of thermal excitations increase rapidly. Unfortunately, in order to make a quantitative comparison of the experimental data obtained by Kapitza with the conclusions of a theory one requires much greater accuracy in the measurements of the various quantities which appear in the equation for the submerged jet.

Section 8. Second Sound

INTRODUCTION. The inertial property of heat, first established in the experiments carried out by Kapitza, implies that there is one other new effect; this effect was predicted by Landau [29] and

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then computed by Lifshits [30], who also considered the conditions most favorable for its observation.

The essence of this effect is as follows. We assume that a certain volume contains the roton-phonon gas with a given particle of density. Suppose now that for some reason or other at a given point in space there is a periodic variation in the density of this thermal gas. Then, specifically because of the inertial property of the heat, the oscillations in the density of the thermal gas propagate in the form of waves which are analogous to the propagation of density variations in an ordinary gas. There is some similarity between the thermal wave or, as it is now called, second sound, and ordinary sound.

In contrast with ordinary sound which is produced by the effect of a varying pressure, second sound arises as a result of a varying temperature (as is well known, a change in the number density of the gas of thermal excitations is completely equivalent to a change in temperature). Whereas temperature variations in ordinary sound take place only as a result of an adiabatic process and thus are small, the pressure variations in second sound arise only as a result of thermal expansion and compression. Since the coefficient of thermal expansion in helium II is very small, the pressure variation is very small and in most cases can be neglected. There is no mass flow in second sound. It is replaced by the flow of heat.

It is obvious that thermal oscillations can be excited in any body. However, in all media with the exception of helium II the thermal wave is attenuated very rapidly. In helium II, because of the inertial property possessed by the thermal flow the thermal waves exhibit essentially no decay.

GENERATION OF SECOND SOUND BY THE THERMAL METHOD. The original experiments by Peshkov [9], in which second sound was first observed, were carried out as follows. A variable voltage of frequency $\frac{1}{2}\omega$ from an audio oscillator is applied to heater 1 (Figure 57), a plane grid of thin constantan wires. The heat generated in the heater is given by

$$W = RI_0^2 \cos^2 \frac{\omega t}{2} = \frac{RI_0^2}{2}(1 + \cos \omega t); \qquad (9.19)$$

in other words there is a constant term and a variable term of frequency ω. The variable heat flow is propagated in the helium II as a nondecaying wave which is accompanied by temperature oscillations.

These temperature oscillations are detected by the resistance thermometer 2 which is made from a phosphor bronze wire 30 μ thick which is wound into a zig-zag helix. The bronze thermometer is fed by the same audio oscillator. However, the frequency of the voltage applied to it is first doubled in a tuned frequency multiplier which is monitored with a cathode ray oscilloscope.

Thus, the voltage in the bronze thermometer circuit is an alternating voltage $E = E_0 \cos \omega t$; on the other hand the resistance varies according to the relation $R = R_0 [1 - \alpha \cos (\omega t + \varphi)]$ at frequency ω and phase φ determined by the temperature oscillations in the thermal wave. Hence, the current in the thermometer circuit is composed of a variable component and a constant component. The magnitude of the constant component $(\alpha E_0 / 2R) \cos \varphi$ depends on the value of the coefficient α, which is a function of the amplitude of the temperature oscillations and the phase difference φ between the temperature oscillations and the alternating-current. The constant current component is measured with the galvanometer g.

When the thermometer is displaced with respect to the heater it is found that the galvanometer reading changes; thus the existence of second sound was demonstrated (cf. Figure 58).

Later experiments on second sound carried out by Peshkov [9] utilized an entirely different system. The measurements are made using standing waves excited in a glass tube 3 (Figure 59), covered at one end by a plane glass disc 10 which causes reflection of the waves generated by heater 7. The latter is wound on the face of a movable disc 6 which, in turn, is attached to tube 4. The thermometer 8 mounted on tube 5 can be moved along the resonator. At the bottom of the resonator there is another thermometer 9. The positions of the heater and thermometer are determined by means of scale 1 and vernier 2.

The electronic system is also different.

The voltage applied to the heater is applied to one pair of plates of a cathode-ray oscilloscope. The thermometers are supplied by
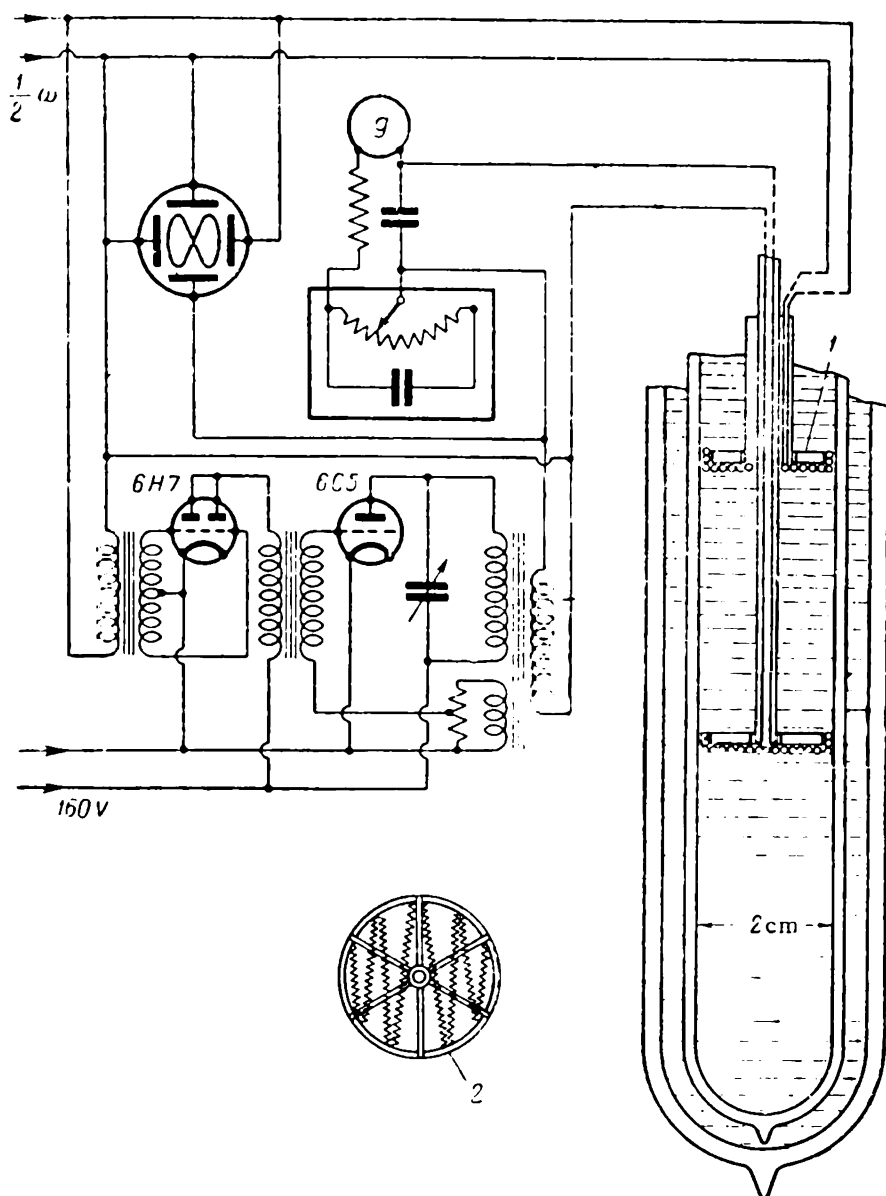
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Fig. 57. Device for observing the propagation of second sound in helium II.

direct current from battery 15 through a high resistance 14. Because of the temperature variations due to the variable heat flow the thermometer resistances and voltage drops across them are periodic functions of time. The potential terminals of the bronze thermometers are connected to amplifier 12 through the switch 13 and then to the second pair of plates of the oscilloscope 11. The galvanometer g in the potentiometer circuit 16 is used to measure the temperature in the helium bath.

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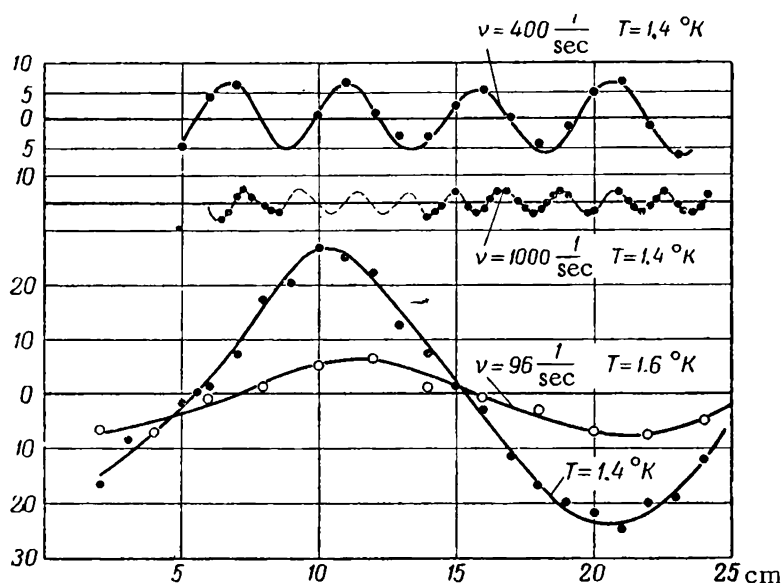
Fig. 58. Galvanometer deflection (in arbitrary units) as a function of the distance between the thermometer and the heater.

In measuring the velocity of second sound the heater is positioned so that the length of the resonator is equivalent to an integral number of half waves. The position of the temperature antinodes is determined with thermometer 8. The velocity of sound u is computed from the formula $u = 2lv/n$ where l is the length of the resonator, n is the number of half waves in a length l and v is the frequency. The accuracy with which Peshkov was able to measure this quantity was 0.3% in the range from 1.3 to 2.0° K and represents the only measurement of this high accuracy for work carried out at such low temperatures. The points at the lower temperatures were taken with a special system in which the method was somewhat modified.

The results of the measurements are shown in Figure 60 and in Table 19.

The maximum on the curve corresponds to a velocity $u = 20.32$ m/sec and occurs at a temperature $T = 1.63°$ K.

The measurements were carried out at frequencies of 200 and 800 cps. At the temperature corresponding to the maximum velocity, measurements were also made at frequencies ranging from 100 to
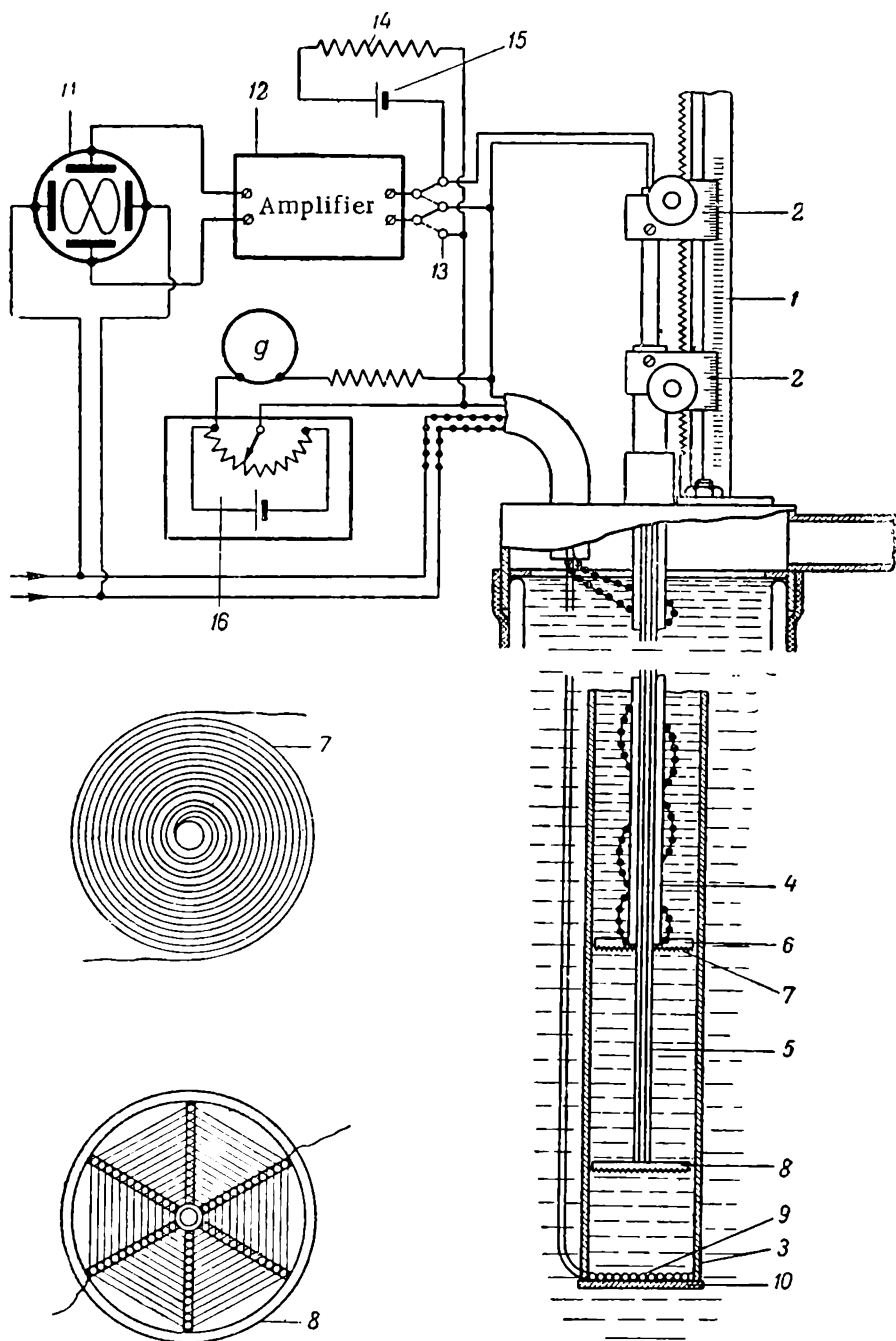
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Fig. 59. System for measuring the velocity of second sound.

10,000 cps in order to measure dispersion. However, no signs of dispersion were found.

Recently Peshkov has carried out work on second sound in the low temperature region [66]. This work, which is of great value for the theory of superfluidity, has established the existence of a clearly
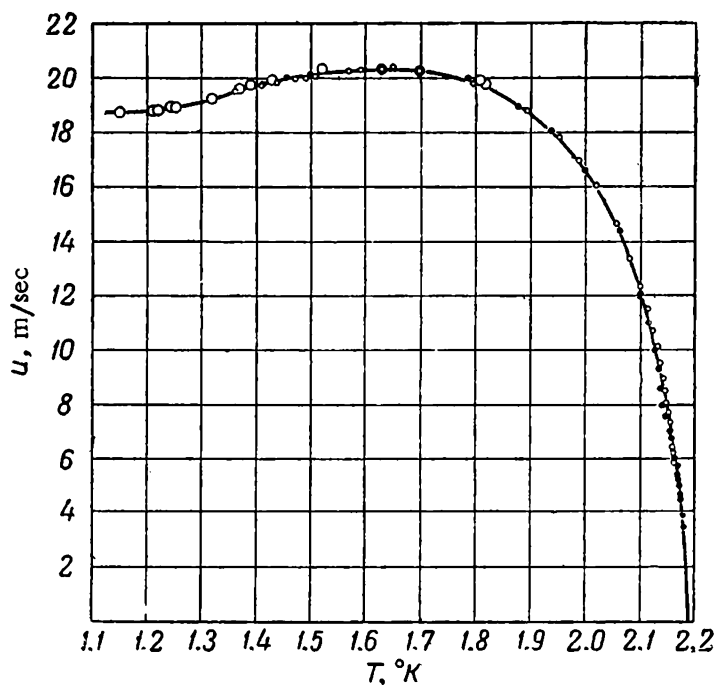
Fig. 60. Velocity of second sound as a function of temperature.

TABLE 19

Dependence of the velocity of propagation of second sound on temperature

(after Peshkov)

T, °K	1.15	1.21	1.25	1.32	1.37	1.41	1.46	1.50
u, m/sec	18.70	18.70	18.95	19.60	19.60	19.73	20.00	20.12
T, °K	1.57	1.59	1.63	1.65	1.70	1.74	1.80	1.88
u, m/sec	20.20	20.25	20.40	20.36	20.30	20.20	19.84	18.88
T, °K	1.90	1.95	2.00	2.06	2.10	2.15	2.17	2.18
u, m/sec	18.78	17.80	16.53	14.56	12.32	8.4	5.7	3.4

defined minimum in the velocity of sound at a temperature of 1.12° K, where the velocity of sound is found to be 18.4 m/sec.

FILTRATION METHOD. In addition to the thermal method Peshkov [31] has used another scheme, in which the thermal wave is excited by a variable flow of "cold" rather than heat. The idea of this experiment is shown in Figure 61.

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A filter with a large number of fine apertures is pressed into a brass cylinder. The helium II which fills the cylinder is subjected to a periodic pressure which is transmitted through a diaphragm.
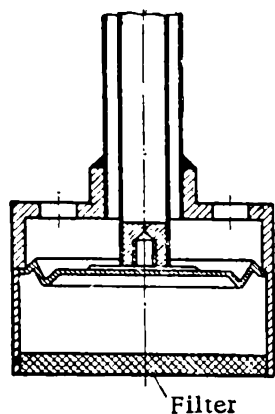

Fig. 61. Arrangement using a filter and radiator for generating second sound by suction through a porous body.

The oscillations of the diaphragm are produced by a device similar to an ordinary loudspeaker coil and are transmitted by means of a steel tube. The periodic pressure variations result in periodic expulsion and suction of the superfluid part of the helium whereas the normal part is more or less filtered by the apertures. This method produces ordinary sound in addition to second sound but the amplitude of the temperature oscillations associated with the second sound is considerably greater than those of the ordinary sound. As in the preceding studies Peshkov used a bronze thermometer for detecting the thermal waves.

Peshkov also carried out experiments to establish the nature of the losses which are characteristic of second sound. An analysis of the frequency dependence of the attenuation led him to the conclusion that the volume losses are negligible and that the predominant role is that played by surface losses due to viscous and thermal effects.

In spite of the fact that the relative velocity between the normal and superfluid parts in the Peshkov experiments was as high as 100 cm/sec and that in a number of cases a strong constant turbulent heat flow was superimposed on the second sound no disturbance of the thermal wave was observed. This result is an indication of the unusual stability of the thermal waves.

CONVERSION OF SECOND SOUND TO FIRST SOUND. The idea of converting second sound to first sound was proposed by Onsager, who also considered the possibility of the reverse process. The first attempts to investigate this effect were made by Lane, Fairbank and Fairbank [32]. The method of exciting second sound used by these authors was very similar to that used by Peshkov [9]. A heater, supplied by an alternating-current source, is placed at the bottom of a cylindrical glass container which is partially submerged in

liquid helium II. The column of liquid between the heater and the free surface is essentially a resonator in which standing waves can be excited by an appropriate choice of frequency. Inasmuch as the surface of the helium II represented an antinode for the temperature oscillations, periodic pressure fluctuations are excited in the layer of vapor adjacent to the surface; these variations propagate in the form of ordinary sound waves (first sound). The pressure variations are picked up by a microphone located above the liquid and the signal from the microphone is fed to an amplifier and then to a recording potentiometer or to the deflection plates of a cathode ray oscilloscope in which the other pair of plates are driven by the alternating-current source used to supply the heater.

Using a cathetometer to measure the drop in the helium level resulting from the gradual evaporation of the liquid and observing the successive appearances and disappearances of resonances at each of the temperatures investigated it is possible to obtain curves such as that shown in Figure 62, Then, using the known frequency and wave length the velocity of second sound can be determined.

The results of these measurements are in good agreement with the curve obtained by Peshkov (cf. Table 20).

TABLE 20

Dependence of the Velocity of Propagation of Second
Sound on Temperature
(after Fairbank, Lane and Fairbank)

T, °K u, m/sec	1.425 19.80	1.453 19.86	1.570 20.23	1.607 20.38	1.685 20.46
T, °K u, m/sec	1.780 20.16	1.795 20.08	1.994 17.2	2.074 14.13	2.15 9.40

It should be noted that the resonance peaks in Figure 62 differ somewhat both in width and height. This effect can be explained by modulation which arises as a result of low frequency oscillations due to propagation of ordinary sound in the helium vapor. The few double peaks are interpreted by the authors as coincidences of resonances in first and second sound propagated in the helium II.
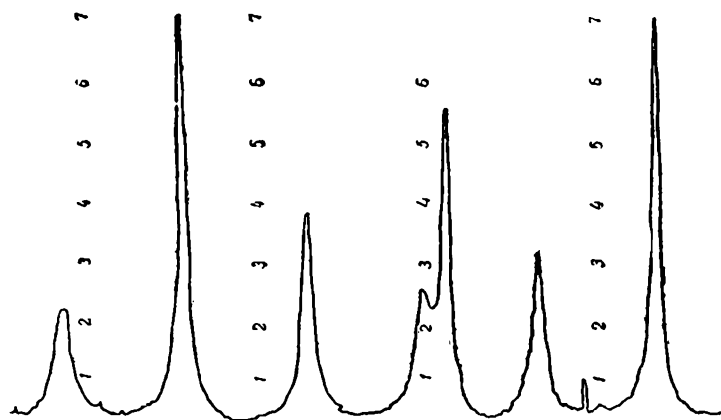
Fig. 62. Second sound resonances obtained with a recording potentiometer at $T = 1.57°$ K. The microphone was located in the vapor above the level of the liquid.

In addition to the technique described above the authors reported another method for detecting second sound in which a microphone is submerged in helium II and in which the measurements are carried out at a fixed frequency but with a varying temperature (slow warming of the cryostat). In this case the appearance and disappearance of resonances is due not to a change in the height of the liquid column but to the temperature variation in the velocity of second sound $u(T) = \dfrac{4\omega_0 L}{n}$, where $u(T)$ is the velocity of sound, $2\omega_0$ is twice the frequency of the heater current, L is the length of the
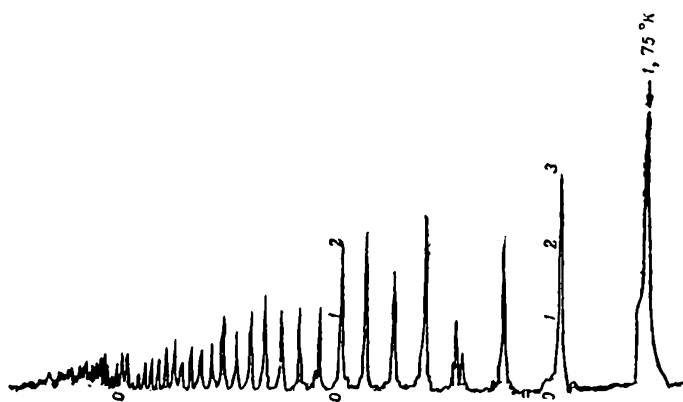

Fig. 63. Second sound resonances obtained with a recording potentiometer with the microphone completely submerged in the liquid.

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resonator and n is a whole number. A tracing obtained with the potentiometer is shown in Figure 63.

The fact that the resonance peaks are so sharp is interpreted by the authors as meaning that the coefficient of conversion and the dissipation factor of second sound are very small.

Attempts to discover the conversion of first sound into second sound, made by Lane, Fairbank, Schultz and Fairbank [33], did not achieve positive results at first. However, in experiments carried out by Peshkov and Belikova [34] the conversion of ordinary sound excited in the vapor into second sound which propagated in the helium II was observed. The first sound was generated in the helium vapor by means of a diaphragm and the second sound was detected in the liquid phase using a phospor bronze thermometer.

The alternation of the resonance peaks was produced by gradual lowering of the level of the liquid which was observed with a cathetometer. As estimate was made of the ratio of the conversion coefficients for the scheme *first sound* $\rightleftharpoons$ *second sound;* this ratio was found to be close to unity.

Reference was also made to the possibility of not only detecting but also generating second sound by means of a diaphragm oscillating in a plane parallel to itself.

Later work by Fairbank, Fairbank and Lane [35] also showed the possibility of converting first sound, generated in the vapor, into second sound, in helium II.

SECOND SOUND UNDER CONDITIONS OF HIGH PRESSURE. In order to measure the velocity of second sound under conditions of high pressure, Peshkov and Zinov'eva [36] used a thick-walled copper cylinder connected to a chamber containing gaseous helium at a pressure of 150 atm. The cylinder was placed in a helium bath and was filled with liquid helium by condensation from the chamber. The temperature inside the cylinder was controlled by the rate of evacuation of vapors from the cryostat surrounding it. Inside the cylinder there was a cylindrical glass resonator at the ends of which were fastened a constantan heater and a bronze thermometer. The frequency applied to the heater was chosen in such a way that standing waves were set up inside the glass tube. The measurement of the velocity of second sound was facilitated by the fact that the

ends of the resonator were the antinodes of the temperature oscillations, i.e., the thermometer was located at the most favorable point. The potential terminals on the bronze thermometer, from which a constant current was taken, were connected through a fine staybright tube with a gasket and applied to the plates of a cathode ray oscilloscope through an amplifier. A voltage from the audio frequency oscillator used to supply the heater was fed to the other pair of plates.

The measurements of the velocity were carried out as follows.

1. $u = u(p)$ curves with T = const. A resonance was established at the required temperature and pressure of the saturated vapor. The subsequent slow increase in pressure made it possible to observe and count off the number of resonances on the screen of the oscilloscope. The velocity of sound was calculated from the length of the resonator, the frequency, and the number of the resonance.

2. $u = u(T)$ curves with p = const. A search was made for a resonance at the required pressure and the lowest temperature which could be achieved. The subsequent slow rise in temperature to the λ-point then made it possible to observe and count the number of resonances on the screen of the oscilloscope. The pressure was regulated by means of a ballast volume. The displacement of the λ-line resulting from the increased pressures was determined from the sharp break in the resistance of the thermometer produced by the sudden change in heat removal conditions in the transition from helium ll to helium l. At all pressures the velocity of propagation of second sound approached zero smoothly as the λ-point was approached. As the freezing curve was approached, however, the velocity remained finite. The freezing curve was taken as the point at which no more resonances were found.

In all these observations the velocity of second sound always exhibits a maximum. A line through the maxima on the curve $p = p(T)$ is parallel to the λ-line. The region in which helium II exists, as plotted by Peshkov and Zinov'eva, is shown in Figure 64. This figure also contains data obtained by Keesom using other methods [37]. Isobars and isotherms for the velocity of second sound are shown in Figures 65 and 66 respectively.

DISCUSSION OF THE RESULTS.   a) *Comparison of the theoretical and experimental results.*   The formula for the velocity of second sound derived by Landau from hydrodynamic considerations is

$$u = \sqrt{\frac{TS^2 \rho_s}{C \rho_n}}.$$ (9.20)

Knowing the experimental values of the velocity of second sound u, the entropy S, and the heat capacity C, using equation (9.20) it is possible to compute the ratio ρ_s/ρ_n at various temperatures. However, as has been pointed out by Khalatnikov [38], the results of the experimental determination of the specific heat C are not accurate enough to allow a comparison of the theoretical and experimental values of ρ_s/ρ_n.

This sort of comparison is made using the specific heat of helium II as calculated from the values of entropy found by Kapitza.

A knowledge of the velocity of second sound and the entropy is found to be sufficient for determining the parameters of the energy spectrum associated with the thermal excitations; the presently accepted values of these parameters are as follows:

$$\frac{\Delta}{k} = (9.6 \pm 0.4)°\text{K}; \qquad p_0 = (2.06 \pm 0.02) \times$$

$$\times 10^{-19} \text{ gm cm/sec};$$
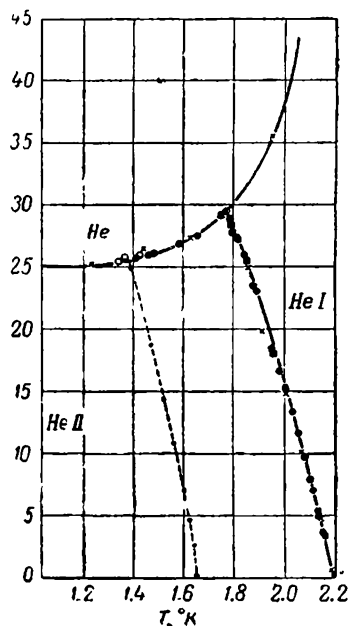
$$\mu = (5.0 \pm 1.5) \cdot 10^{-24} \text{ gm}$$

Fig. 64. Boundaries of the helium II region as obtained from the velocity of second sound. The pressure is plotted in atmospheres along the ordinate axis.
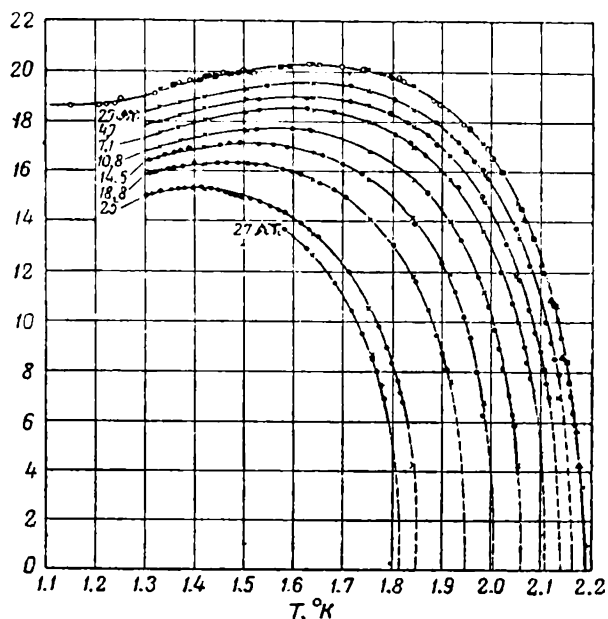
The poor accuracy assigned to the determination of μ and Δ reflect the inconsistencies in the presently available experimental data.

Studies of the velocity of second sound, especially at low temperatures, are of special importance for the theory since they make it possible to determine the magnitude of the parameters. Moreover, the minimum at a temperature of 1.12° K on the $u = u(T)$ curve,

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Fig. 65. Isobars for the velocity of second
sound. The numbers along the ordinate
axis refer to u in m/sec.

which was discovered by Peshkov, corroborates the results of later
measurements of the viscosity of the normal part of helium II [12];
both of these measurements are important in that they indicate the
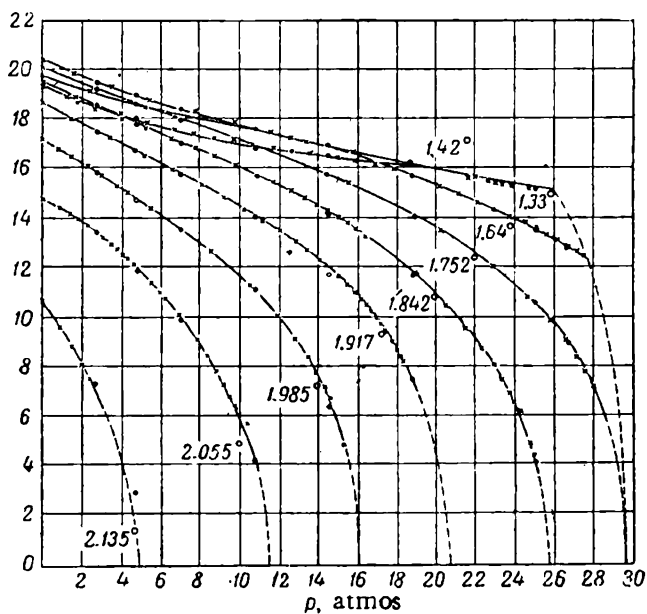validity of the Landau theory and the erroneous results of the Tisza
analysis.

Fig. 66. Isotherms for the velocity of second
sound. The numbers along the ordinate axis
refer to u in m/sec.

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The investigation of the velocity of second sound under conditions of high pressure carried out by Peshkov and Zinov'eva [36] also are important for the development of the theory of thermal quanta since this work allows us to determine the variation in the values of the spectral parameters as the density of helium II varies. The data which were obtained have been presented by Khalatnikov [38] in his analysis of the scattering of thermal excitations on each other. As has already been mentioned in Section 3, this theory yields very good agreement with the experimental results obtained by Andronikashvili [12].

b) *Thermal shock*. The velocity with which a thermal pulse is propagated in helium II was considered by Ganz [39] in a paper which has been discussed in Chapter VI. The results obtained by Ganz were analyzed in detail by Peshkov [26]. In work in which the thermal shock was produced by the discharge of a condenser Ganz apparently detected the passage of the ordinary sound wave, the effect of which was manifest in the formation of gas bubbles due to boiling of the liquid. However, the propagation of first sound in a very fine capillary or in a film is subject to peculiar effects if one recalls the filtration effect described in Section 4. Since the pressure oscillations of the normal part must be damped by friction with the walls of the narrow capillary the pressure oscillations can apparently be transmitted only by the superfluid. This situation, in turn, must lead to a considerable variation in superfluid concentration and, consequently, temperature. This type of temperature variation could be transmitted with the velocity of ordinary sound. It is very possible that it is this effect which was detected by Ganz.

On the other hand, in the experimental results shown in Figure 162 of *Helium* by W.H. Keesom (in which ordinary sound could not be produced) the time required for the travel of the thermal pulse is greater by one order of magnitude. Peshkov called attention to the fact that in these experiments the velocity of propagation of the thermal pulse is found to be approximately 20 m/sec, a value which is in good agreement with the other data on the velocity of second sound.

The conclusion reached by Ganz on the basis of his own experiments was that heat is propagated in helium II with the velocity with which the atoms move at these temperatures; this conclusion is undoubtedly in error.

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Section 9. Films

FILM THICKNESS[2]. In addition to the experimental results which have been presented in Section 5 of Chapter VI brief mention should be made of certain new data concerning the thickness of the film. As is well known, in the earlier experiments the film thickness was estimated at approximately $3 - 5 \cdot 10^{-6}$ cm where the measurements referred to an average thickness of the layer on the solid surface, extending above the level of the liquid to distances of 15 - 20 cm.

The work carried out by Atkins [40], is based on the effects already known from the experiments by Allen and Misener [20]. The essence of these effects is as follows. In the flow of helium II from one container into another through a system of very narrow parallel capillaries, as both levels approach each other there is a slowly decaying harmonic oscillation of the liquid about the equilibrium position.

A similar effect has been observed by Atkins when the two containers are connected by a wall film rather than a narrow capillary. The oscillation period is found to be related to the film thickness in the fairly simple fashion.

Another method consists of measuring the amount of liquid which flows over barriers of different height in the absence of a pressure differential.

In this way Atkins has been able to establish the fact that the thickness of the film is inversely proportional to its height above the level of the liquid; this result is in agreement with the results obtained with the first method. Actually, in both cases it is necessary to make certain assumptions with respect to the nature of the flow.

A direct determination of the film thickness by an optical method was made by Jackson and Burge [40]. A beam of polarized light is directed onto the surface of a metal, part of which was covered by a transparent monomolecular layer of barium stearate. A transparent trimolecular layer of the same material is deposited on the

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[2]The material in this section is taken from a survey paper by Mendelssohn [40] which contains unpublished work by English investigators.

other part. The reflected waves passes through a quarter-wave plate and a Nicol prism. The illumination of both fields is equalized by choosing an appropriate position for the Nicol prism. When the layer of barium stearate is covered by a film of helium II the equalization of the illumination is disturbed and can be restored by rotation of the Nicol prism through some angle, from which the film thickness can be computed.

The results of preliminary observations are in agreement with the data measured by other authors.

VAPOR PRESSURE ABOVE THE FILM. All experiments which have been carried out to this point for the purpose of studying the properties of wall films were carried out under conditions in which the film was under a vapor pressure in equilibrium with the free surface of the liquid. Brown and Mendelssohn [41] investigated the behavior of a film which was isolated from the effect of the vapors above the free surface. The apparatus used by these authors is shown schematically in Figure 67 a.

A resevoir containing liquid helium is divided into two sections — A and B. The vapors from B can pass into A through a system of narrow capillaries L similar to those used by Allen and Misener [20] in studying the viscosity of helium II. The measurement of the vapor pressure and the evacuation from A and B are carried out independently.

Fig. 67. System for studying the properties of wall films of helium II at reduced vapor pressure.

If the liquid in B is at a temperature lower than the λ-point, then the gas flow through L is supplemented by film flow. Using a system of capillaries with successively smaller openings it is possible to make the gas flow as small as desired compared with the film flow.

The original experiments were devoted to an investigation of the equilibrium pressure of the vapor above the film. It was found that because of the thermo-mechanical effect in the film, the pressure of the vapor in the upper container depended on small variations in
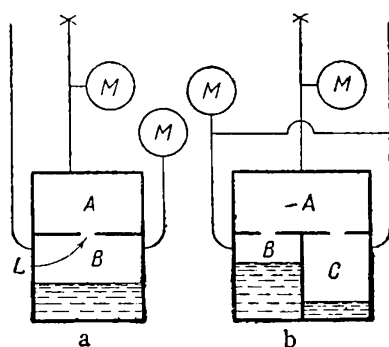
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the difference of temperature between A and B, and under certain conditions could exceed the vapor pressure in B by a factor of 2. The question of whether "bulk" liquid was formed from the film under these conditions was not definitely answered. Also, no definite conclusion was reached as to the possible existence of a temperature gradient in the film itself.

By dividing the lower reservoir in two as shown in Figure 67 b and by introducing a second system of capillaries between A and C it is possible to observe the flow of the film through a space in which the vapor pressure is lower than the equilibrium pressure above the "bulk" liquid. One of the features of this flow is the difference in levels between B and C.

THERMO-MECHANICAL AND MECHANO-CALORIC EFFECTS IN FILMS. In addition to the work which has been described in Section 5 of Chapter VI mention should be made of the results obtained by Strelkov [19] independently of the work done by Daunt and Mendelssohn [42], [43].

Using a weak beam of light to heat one end of an inverted U-tube half filled with helium II (Figure 68) Strelkov found that the wall film of the liquid moved quickly from the cold end to the heated end. This effect may by considered a direct analogy of the thermo-mechanical effect observed in the flow of helium II through capillaries.

Fig. 68. Device for studying the "inertial" motion of films.

Strelkov observed the siphon effect (the inverse of the thermo-mechanical effect), which is the analog of the mechano-caloric effect, both by direct count of the drops of helium which flowed over the bend in the tube as well as direct observation of the level of the liquid in the tube. On the basis of these experiments Strelkov concluded that the product vd of the flow velocity v and the film thickness d is only weakly dependent on the height of the siphon (the distance taken upward from the level of the liquid to the bend in the tube). Thus, the flow of liquid in the film, as in flow in a thin capillary, is essentially independent of pressure.
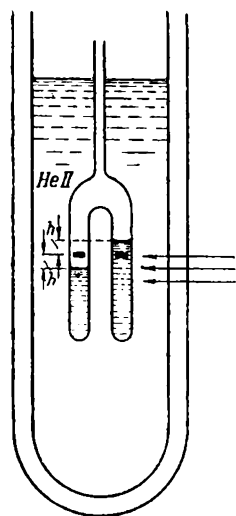
If the difference in levels is large, a small increase in the velocity of the flow is observed only when one of the levels of the liquid is in direct proximity to the bend in the tube. The author explained this effect by the dependence of film thickness on its height above the level of the liquid.

A strong dependence of the quantity vd on pressure can be observed when the levels are fairly close — of the order of 0.5 mm. This effect is independent of the distance from the liquid levels to the bend in the tube.

Strelkov obtained a temperature dependence for the quantity which is completely similar to the curve in Figure 178 of *Helium* by W.H. Keesom.

MOTION OF A FILM WITHOUT GRAVITATIONAL FORCES. A peculiar motion in surface films has been observed by Daunt and Mendelssohn [44]. In this work a tube O consisting of two concentric containers 1 and 2 is half submerged in a helium bath B (Figure 69). After a certain period of time both containers become filled with helium II and all three levels are the same. If the tube is raised above the level of the liquid in the bath both c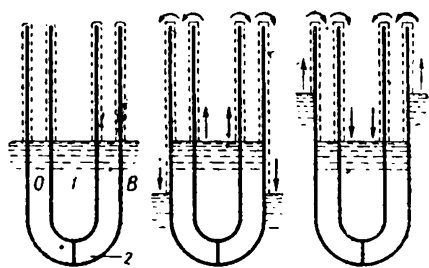ontainers, 1 and 2, start to empty. Since the amount of liquid which flows over the rim of the container by means of the film is determined by its perimeter, container 1 empties more rapidly than container 2; thus the levels of the liquid in these containers become different in the course of time. The experimental results indicated, however, that to an accuracy of 0.1 mm the levels of the helium in both containers remained unchanged with respect to each other.

Fig. 69. Device for studying the "inertial" motion of films.

In the opinion of the authors, it is apparent that the helium flows from container 2 while under the influence of gravitational potential effects. The flow of liquid from container 1 into container 2 is free from gravitational potential effects. On this basis the authors estimate the linear flow velocity from container 1 into container 2;

this can be computed knowing the liquid flow rate and the thickness of the film.

The film velocity in the experiments performed by Daunt and Mendelssohn apparently reached 18 cm/sec. The authors argue that to maintain this velocity by means of gravitational acceleration would require a difference of at least 1.6 mm in the levels in containers 1 and 2.

A similar picture is found when the level of the helium II in both tubes is below the level of the helium in the surrounding bath: the film creeps from the bath into tube 2 and from it into tube 1 with velocities such that the levels of the liquid in containers 1 and 2 do not change with respect to each other.

The authors propose that under these conditions nonviscous motion of the film takes place by inertia.

MOMENTUM OF THE FILM. Strelkov [19] has made an interesting attempt to measure the momentum of a helium film. For this purpose he suspended light weights by a spring; a thin ribbon made from foil was suspended from the weights. The lower part of the ribbon was submerged in helium II. The heat load was applied to it by means of a weak beam of light. The author expected that the film of helium II, in creeping up the ribbon against the temperature gradient, would impart momentum of the opposite sign to the ribbon, i.e., that it would cause the spring holding the weights to be stretched. In some of the experiments the ribbon was replaced by a wire which was furnished with superconducting leads. In this case the heat load was applied by an electrical method.

The results of several experiments showed that the creeping of the film pushed the heated body up, forcing the weight holding the springs to be compressed rather than stretched, as had been expected.

However, this behavior is actually quite reasonable. As is well known, the superfluid mass, which is carried by the film, can only execute potential motion in which state it cannot exert a pressure on a streamlined body; whence it follows that it can not exert a reactive force on the weights used by Strelkov. However, in being converted to a normal mass under the effect of heat it transfers its

momentum to the latter. Because of frictional forces the normal mass is slowed down rapidly and gives up momentum to the spring weights, which are then pushed out of the helium.

It is completely understandable that this explanation would be possible only on the basis of the theory of thermal motion in helium II which was proposed several years after the experiments carried out by Strelkov.

THE PROBLEM OF FILM FORMATION FROM THE GAS PHASE. The possibility of film formation from the gas phase has been considered by Mendelssohn and Daunt [45].
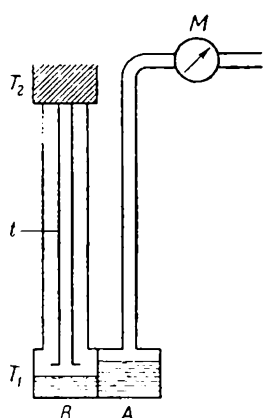

Fig. 70. Diagram of a device used for studying the condensation of films from vapor.

A diagram of the experiment is shown in Figure 70. Container A contains helium under reduced pressure at a temperature T_1. Container A is in thermal contact with a reservoir at temperature T_2. The thermal contact is realized through the metal wall which touches the liquid helium partially filling container B and through the wall of container B, the upper part of which is at the temperature T_2. Inside B there is a system of unsilvered tubes t which terminate in a horizontal flange at the bottom. The upper ends of the tubes are in contact with the resevoir at temperature T_2. By changing the level of the helium in B the flange can be submerged in the liquid or raised above the liquid. The heat transfer through the walls of container B and tubes t takes place chiefly as a result of the motion of the film and depends on the amount of film on the surfaces of tubes t. From the amount of heat taken from the reservoir at T_2 it is possible to estimate the amount of helium evaporated from A. For this purpose the evacuation system is provided with an arrangement for measuring the volume of gas which flows through it.

From these experiments it is found that the ratio of the amount of heat conducted from T_2 to T_1 when the flange is submerged in the helium to the amount of heat which flows when the flange is above the surface of the liquid is 1.95. Under these conditions the ratio of perimeters "wall plus tube/wall" is 1.85. Thus, it is clear

that within an accuracy of 5% the heat conduction by the film is proportional to the perimeter of the surface in contact with the liquid helium ll. Appropriate corrections are made for heat conduction through the gas phase and the wall materials.

From these experiments the authors conclude that the film at the surface of the tube is formed chiefly when the latter is in contact with the liquid. If deposition of the film does occur from the gaseous phase, in any case it is in an amount which does not exceed 10% of R, where R is the amount of film transported with the critical velocity.

It should be noted that in order for this conclusion to be correct one must assume zero thermal resistance for the thin layer of vapor which separates the flange from the surface of the liquid.

Section 10. Impurities

BEHAVIOR OF COLLOIDS IN HELIUM II. Savich and Shal'nikov [46] have found a basic difference in the behavior of colloidal particles suspended in helium 1 and in helium ll. Gaseous helium, containing a mixture of air or hydrogen, was admitted to a dewar containing liquid helium. The impurities were condensed and a colloid of fine crystals of the solidified gas was formed. In helium 1 the colloidal air (or hydrogen) was completely stable, yielding a characteristic opalescence when illuminated from the sides. However, as soon as the temperature dropped below the λ-point the colloid coagulated rapidly and precipitated out in the form of coarse, porous flakes. The origin of this effect is as yet unknown.

THE He^3 ISOTOPE. Progress in the physics of liquid helium and the techniques of separating isotopes have made it possible to solve the problem of extracting the isotope He^3. An investigation of the properties of He^3 is of importance both for nuclear physics as well as an understanding of superfluidity. He^3 can be investigated in a cyclotron. An exact determination of the mass of the He^3 nucleus is of great theoretical interest. As is well known, no substance can be dissolved to any appreciable extent in liquid helium; the only exception in a solution of He^3 in He^4. Finally, an investigation of the properties of He^3 with respect to superfluidity is of greatest importance.

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The He^3 content in helium is very small: mass spectrometer measurements [47], [48] indicate that the ratio of the number of He^3 atoms to the number of He^4 atoms in helium taken from the atmosphere is $1.1 \cdot 10^{-6}$, while in helium obtained from mineral sources this ratio varies from $0.2 \cdot 10^{-7}$ to $120 \cdot 10^{-7}$.

The extraction of an isotope of such small concentration is obviously a problem of great difficulty and for this reason attempts have been made to exploit the special properties of liquid helium for this purpose, especially the superfluidity property.

SEPARATION OF HELIUM ISOTOPES BY CRYOGENIC TECHNIQUES. As is well known, London and Tisza (cf. Section 1, Chap. VIII) associated superfluidity with a degenerate Bose-Einstein gas. On this basis, Franck [49] concluded that since the atoms of the He^3 isotope do not obey Bose-Einstein statistics they should not exhibit the super-fluidity property.[3] Franck made the suggestion that it might be possible to separate the helium isotopes by filtration. Franck's argument was criticized on the basis of the Landau theory by the author of the theory and Pomeranchuk [50]. Landau and Pomeranchuk argued that since superfluidity is not a property of individual atoms but a collective effect for identical He^4 atoms, any other atoms comprising small impurities, such as He^3 or the rarer isotope He^6, would not partici-pate in the superfluid motion regardless of whether or not they themselves possess the superfluid property.

Fig. 71. Device for se-parating the isotopes He^3 and He^4 by filtra-tion through a film.

Daunt, Probst, Johnston, Aldrich and Nier [51] have investigated experimentally the possibility of separating these isotopes on the basis of superfluidity. In this work use was made of the apparatus shown in Figure 71. In container A containing liquid helium taken
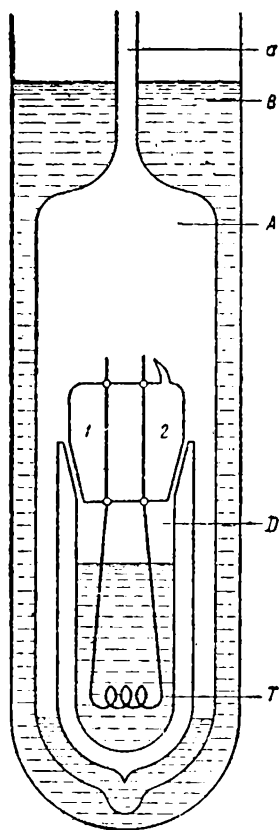
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[3]Similar ideas were advanced by Onsager (cf. Rev. [53]).

from the atmosphere there is a small dewar D which is partially submerged in the liquid helium; the aperture in the dewar is closed by a ground glass stopper which hinders the penetration of helium vapor from container A into the dewar D. The dewar contains an electric heater T. The whole system is placed in a helium bath B at a temperature 1.3° K. When current is passed through the heater, superfluid flow in the form of a surface film is created (cf. Section 9 of this Chapter); this flow proceeds smoothly through the gap in the cross section in the small dewar. A mass spectrometer investigation of the contents of the dewar shows that He^3/He^4 ratio in it is less than $5 \cdot 10^{-8}$ whereas the initial He^3/He^4 ratio is $1.2 \cdot 10^{-6}$. In this way it was demonstrated that the He^3 does not participate in the superfluid motion. The authors hoped to find an increase in the He^3 concentration in the helium vapors in container A during the course of the experiment. However, it was found that in samples of the vapor taken through tube a the He^3/He^4 ratio was smaller than that of the original helium by approximately a factor of three; in other words, a large part of the He^3 was in the liquid helium. This reduction of the He^3 content in the vapors above the liquid helium which is below the λ-point may, as indicated by the authors, either be a new property of helium or be explained by a disturbance of the equilibrium distribution of isotopes in the vapor due to the strong evaporation of He^4 from the surface film in tube a and convective flow of the vapors back into container A.

In later work, Daunt, Probst and Johnston [52] condensed the helium in container A (Figure 71) above the level of the stopper, realizing filtration of the liquid helium through the narrow slit (width of the order of 1 μ) and again it was found that the helium in container A was enriched in the He^3 isotope. The increase in the He^3 concentration outside the dewar D was approximately a factor of 4 or 5 in both series of experiments carried out by Daunt and his colleagues.

Lane, Fairbank, Aldrich and Nier [53] were successful in affecting a considerably increased separation of the isotopes by setting up a thermal flow in bulk liquid helium. As is well known, a thermal flow in liquid helium is realized by the opposing flows of the normal and superfluid components (cf. Section 2, Chap. VIII). Under these conditions however the macroscopic density flow can be zero. Inasmuch as the He^3 can participate only in the motion of

the normal component, in cases of free convection this isotope will be concentrated at points of low temperature.
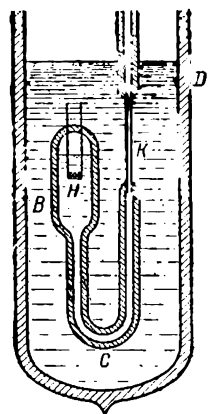

Fig. 72. Device for separating the isotopes He^3 and He^4 by the thermal-flow method.

Figure 72 shows the apparatus used by Lane et al. A thick-walled glass container B with an electric heater H is connected by means of a glass capillary C to a metal capillary K (of kovar) and then to a tube which projects beyond the cryostat. When current is passed through the heater the heat flows along capillary C into capillary K and then through the metal walls into the helium bath. To determine the He^3 concentration in capillary K the authors took samples of gaseous helium above the surface of the liquid in K. In the course of time, at a temperature of 2.01° K the He^3 content in the vapor increased gradually and after 45 minutes was 130 times larger than the original. On the basis of this result the authors suggest that it should be possible to build a system for practical separation of the helium isotopes using a method based on thermal flow.

On the other hand, at a temperature of 1.83° K there is a considerable reduction in the He^3 content in the vapor regardless of the magnitude of the heat flow.

DISTRIBUTION OF THE He^3 ISOTOPE BETWEEN TWO PHASES OF THE SOLVENT. In order to obtain a successful solution to the problem of separating He^3 and He^4 it was found necessary to investigate the distribution of the He^3 isotope between the liquid and gaseous phases of the solvent at various temperatures. This problem was investigated in two papers by Fairbank, Lane, Aldrich and Nier. In the first paper [54] the temperature dependence of the ratio of the He^3 concentrations in the vapor and in the liquid was investigated in the temperature region from 2.2 to 4.2° K. In the second [55] the temperature range was extended to 1.7 and 5.2° K. The authors used a system consisting of a glass flask which was submerged in a helium bath; the helium being investigated condensed in the flask. Using a tube which extended out of the flask samples of the helium vapor were taken by admitting the vapors into a previously evacuated small container connected to the tube. The volume of the container

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was smaller than the volume occupied by the vapor in the flask but larger than the volume of the tube. Using a mass spectrometer the ratio C_v was determined for the He^3 and He^4 isotopes in the vapor. Knowing C_v it was possible to compute the relative He^3 content in the liquid C_l using the mass balance equation for the isotopes. In order to avoid effects on the measurements due to vapors in the connecting tube left over from previous measurements two samples were taken at each temperature. Before taking a sample the cryostat was kept at a fixed temperature above the λ-point for 30 minutes and then below the λ-point for 15 minutes.
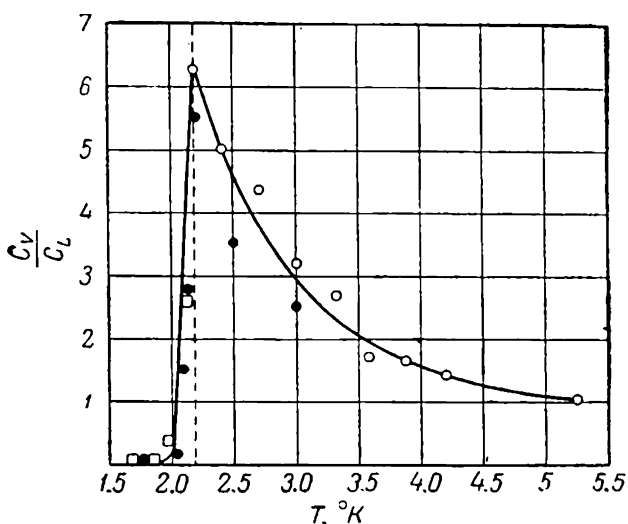

Fig. 73. Relative concentration of He^3 in vapor and in the liquid as a function of temperature.

Special attention was given by the authors to evaporation from the surface film, this being the most likely source of error. The surface film was raised from the flask by a connecting tube to a level at which the temperature was 2.19° K and evaporated; as a result a vapor flow was created (consisting exclusively of He^4) back into the bowl and this disturbed the equilibrium distribution of the isotopes. Since the amount of helium carried by the film is proportional to the perimeter of the connecting surface (cf. Section 5, Chap. VI) the connecting tube was a capillary 1.5 mm in cross section. In order to avoid partial evaporation of the film at the walls of the flask before entrance into the capillary, the bowl was shielded. The authors computed the amount of helium evaporated from the

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film during the time which elapsed between taking samples and indicate that even under the most unfavorable conditions (all of this helium entering the sample) the true value of C_v/C_l does not differ from the measured value by more than 10%.

In Figure 73 are shown the results obtained by Fairbank et al. The authors used C_v/C_l as the most convenient and descriptive index since, in accordance with Henry's law for dilute solutions, this quantity is independent of the total concentration for a given temperature.

In the opinion of these authors the results of this work indicate a simple method for enriching helium in the He^3 isotope: namely, the evaporation of helium at temperatures considerably below the λ-point.

On the basis of the results obtained by Fairbank et al., London and Rice [56] and Stout [57] attempted to formulate a theory for the equilibrium of two phases in a weak solution of He^3 in He^4.
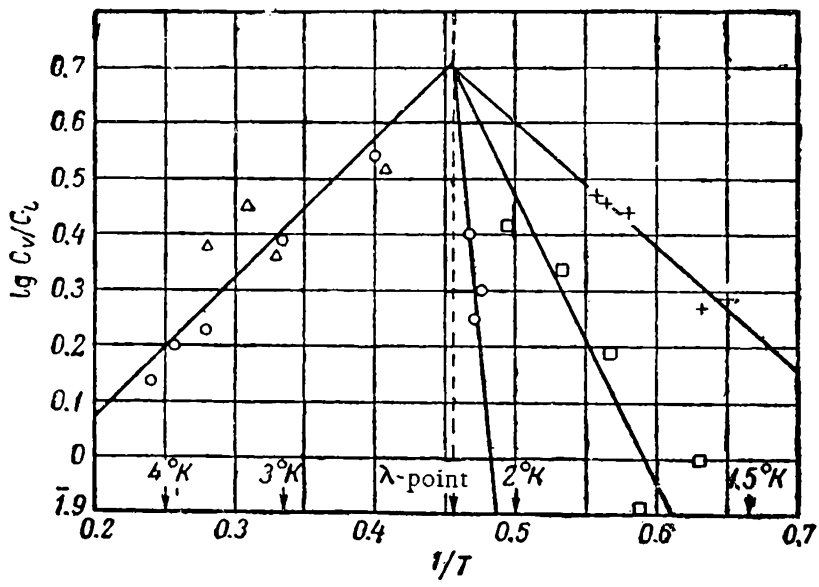

Fig. 74. Dependence of the logarithm of the ratio of the He^3 concentration in the vapor to that in the liquid as a function of $1/T$ after Daunt et al.

The results of a later experiment [58], carried out by Daunt, Probst and Smith, are shown in Figure 74. The reciprocal tempera-

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ture $1/T$ is plotted along the abscissa axis and values of lg C_v/C_l are plotted along the ordinate axis. In the helium I region the experimental points lie approximately on one curve; in the helium II region, on the other hand, each concentration is characterized by its own curve. The circles indicate the results taken from the work of Fairbank et al. [55] (Figure 73) with a total concentration $He^3/He^4 = 1.2 \cdot 10^{-6}$; the squares and crosses indicate the results obtained by Daunt et al.; the squares refer to a total concentration of $10 \cdot 10^{-6}$ while the crosses refer to a concentration of $30 \cdot 10^{-6}$. These results are in disagreement with Henry's law.

The paper by Daunt et al. did not contain a description of the apparatus or a description of the experimental method; the authors note, however, that measures were taken to prevent spurious effects due to the surface film.[4]

Rollin and Hatton [59] measured C_v/C_l for helium for which the initial concentration was $He^3/He^4 = 1.5 \cdot 10^{-3}$ and found that at 1.3° K the relative He^3 content in the vapor was 10 times larger than in the liquid.

DISCUSSION OF THE RESULTS. It follows from the work of Fairbanks et al. [55], Daunt et al. [58], and Rollin and Hatton [59] that the relative He^3 content in the vapor below the λ-point does not remain fixed but increases as the total He^3 concentration is increased. This means that the results disagree with Henry's law even at concentrations of the order of $He^3/He^4 = 10^{-6}$. It should be recalled that Henry's law is derived from thermodynamic equations under the single assumption that the solution is dilute (cf. Ref. [60]). Departures from Henry's law appear with the onset of interaction effects between the molecules of the dissolved substance. It is impossible to to believe that there is any sizable interaction between the He^3 atoms
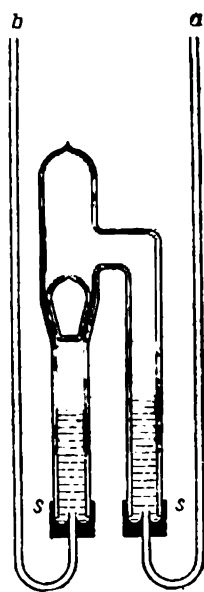

Fig. 75. Device for studying the osmotic pressure of the He^3 isotope.

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[4]In their communication Daunt and his colleagues indicate that in a device built for separation of the isotopes the film is capable of transporting 0.5 liters of liquid helium per hour.

at concentrations of the order of 10^{-6}. Whence it follows that Henry's law must be satisfied for an equilibrium distribution of He³ between two phases of the solvent. On the other hand, the existence of an equilibrium distribution in the helium II region is not a trivial point — equilibrium is disturbed by virtue of the motion of the film and by convection currents in the liquid which arise as a result of local overheating. Hence, one must agree with Rollin and Hatton [59] that in the work of Fairbank et al. [55], in spite of all pre-cautions and estimates, the effect of the film has not been alto-gether taken into account. The smaller the total He³ concentration, the larger is the He⁴ excess created by the film and the smaller will be the measured relative He³ content in the vapor.

Obviously, the theoretical work in Refs. [56] and [57], which is based on incorrect results, is also in error.

OTHER PROPERTIES OF A SOLUTION OF He³ IN He⁴. In addition to the problem of the distribution of the He³ isotope between the two phases of the solvent, other properties of a solution of He³ in He⁴ have been investigated; specifically these are the osmotic pressure due to the He³ in solution and the change in the helium vapor pressure due to the presence of the He³ isotope. It should be noted once again that these two effects have not been investigated quantitatively as a function of various factors — only their existence has been demonstrated.

The first effect was discovered by Daunt, Probst and Johnston [61]. The apparatus used by these authors (Figure 75) consists of a Π-shaped tube. In the upper part of the tube there is a glass stopper designed to prevent the flow of helium vapor from one column into the other. The ends are connected to tubes which extend beyond the cryostat; helium is admitted into the system through these tubes. The device is submerged in a helium bath at a temperature of 1.57° K. At one of the ends there is condensation of helium, the initial concentration of which is He³/He⁴ 2.4·10⁻⁵. He⁴ in the form of a film flows into the other end. However, the levels in the columns are not equalized. The level difference under these conditions is found to be 2.1 mm. The authors indicate the difficulty in inter-preting this result since He³ is contained in both the liquid and the vapor and in different concentrations.

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The change in vapor pressure due to the He^3 isotope was found by Fairbank, Reynolds, Lane, McInteer, Aldrich and Nier [62]. These authors used a system consisting of two identical thick-walled glass containers connected by tubes to the two ends of differential oil manometer. The containers are immersed in a helium bath. One
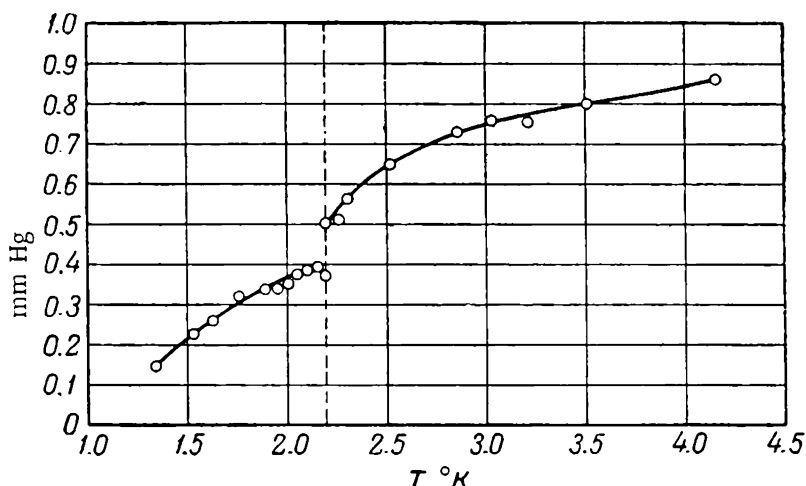

Fig. 76. Temperature dependence of the change in helium vapor pressure due to the He^3 impurity after Fairbank et al. [62].

of the containers holds ordinary helium from natural sources $(He^3/He^4 = 1.6 \cdot 10^{-7})$, and the other contains enriched helium $(He^3/He^4 = 1.6 \cdot 10^{-3})$. In Figure 76 are shown the results obtained in this work. The authors note that the break at the λ-point was reproduced in all measurements.

SEPARATION OF HELIUM ISOTOPES BY THE THERMAL DIFFUSION METHOD. Jones and Furry [63] have carried out a calculation for helium separation in a thermal diffusion system. The authors note that the calculation is based on tables and formulas which do not apply to helium and which, for this reason, must be considered approximate. According to this calculation the system should yield 3.8 cm^3 of helium per day with $He^3/He^4 = 10^{-2} - 10^{-3}$ with a start-up time of 14 days.

McInteer, Aldrich and Nier [64] constructed a system similar to that proposed by Jones and Furry. It consists of a series of two columns with concentric tubes and one column with a heater wire.

According to the authors' measurements this apparatus, with a start-up time of 29 days, yields 7 cm^3 of helium (at standard conditions) the concentration of which is $He^3/He^4 = 4.2 \cdot 10^{-3}$, if the initial concentration is $1.15 \cdot 10^{-7}$ (enrichment of 36,600). It is possible to increase the enrichment by reducing the daily yield of enriched gas. In this case the time required to reach operating conditions is also increased.

Andrew and Smythe [65] have built a system in which a Clusius-Dickel column (column containing a heater wire) is in series with a Hertz column (which is based on the difference of flow velocities through a mercury vapor jet for the isotopes). A sample of mass approximately 0.05 mg is obtained after two weeks of operation and contains 0.5% He^3.

It should be noted that in the majority of cases considered above in which enriched helium is used this helium is obtained by a thermal diffusion method.

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